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FLOW THROUGH WEIR NOTCHES WITH THIN EDGES AND FULL CONTRACTIONS[1]

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CONTENTS

INTRODUCTION

The developments in irrigation agriculture in the arid West have caused many changes to be made in the method of delivering water to canals and to individual irrigators. The value of water increases with the increase of irrigated acreage, and the long-accepted practice of fixing the charges for water on a per-acre-per-annum basis is rapidly losing ground in favor of charges based on the volume of water delivered. When irrigators pay according to the amounts of water used, there is every incentive for them to study the water requirements of their crops and to use the least quantities they judge to be necessary. This leads to a proper economy in the use of water, permits a greater acreage to be irrigated with the available water supply, and conserves the land.

The transition from a flat rate to a rate based on the water actually used is calling for a better knowledge of the accuracy and practicability of existing measuring devices as well as the development of new devices. The weir is generally considered an accurate device for measuring water, and it doubtless is such, provided it is properly installed and the correct formula is used for determining the discharge through the notch. Weirs constitute a large proportion of the devices in use for measuring irrigation water at the present time, being principally of the rectangular notch

[1] This paper is based on experiments conducted in the hydraulic laboratory at Fort Collins, Colo., under cooperative agreement between the Office of Experiment Stations of the United States Department of Agriculture and the Colorado Agricultural Experiment Station.

or Francis type, and the Cipolletti type. Most of the weirs in use have notches with crest lengths of 4 feet or less, being such as are adapted to the delivery of water for farm units. Unfortunately, owing chiefly to the confusion of the statements contained in the literature on weirs, various standards of dimensions have been used in the construction of the weirs now in use. This lack of uniformity results in many erroneous measurements.

The basic experiments with notches having thin edges and full contractions were made by James B. Francis (5)[1] from 1848 to 1852. These have subsequently been enlarged upon by several experimenters and mathematicians. Francis made three series of experiments with rectangular-notch weirs, but the discharges were measured directly in only one series (5, p. 75–76). In each of the two other series an equal flow of water was made to pass through notches of different lengths, the crest lengths and the heads being noted. In the experiments, where the discharges were measured volumetrically, only notches of approximately 8- and 10-foot lengths were used, and the heads ranged from only 7 to 19 inches (5, p. 122–125). Most of the experiments were made with the 10-foot notch, as they were to be applied directly to the measurement of water for power purposes. Francis stated (5, p. 133) that the formula which he derived would apply to heads ranging from 6 to 24 inches, but in no case was it to be used either for heads exceeding one-third the length of the crest or for very small heads. With these limitations the formula can not be used for weirs having crest lengths of less than 1.5 feet nor for heads exceeding 2 feet. For a 1.5-foot crest the formula can be used only for a 0.5-foot head. Horton states (7) that the Francis data and formula will hold for heads from 0.5 foot to 4 feet. Francis's experiments were very carefully and conscientiously made, but were with longer notches and greater volumes of water than are usually needed in delivering water to irrigators. The Francis formula is frequently used, however, without regard to the limits which he imposed upon it, and it is not uncommon to see tables computed from it that give discharges for heads as low as 0.01 foot, with heads as high as 1 foot for a crest length of 1 foot, and for crest lengths varying from 0.5 foot to 20 feet.

The most popular weir notch has been the trapezoidal type with side slopes of one horizontal to four vertical. This type was designed and the formula deduced by the Italian engineer Cesare Cipolletti (3), with the idea of automatically eliminating the correction for end contractions necessary with the rectangular notches and thus obtaining a type of notch the discharge through which would be proportional to the length of the crest and free from error in excess of one-half of 1 per cent from any single cause. Cipolletti derived the shape of the notch by a mathematical modification of the Francis formula for the rectangular notch.

[1] Reference is made by number to "Literature cited," p. 1112–1113.

He obtained the values for the coefficient and exponent by examining Francis's experimental data and increasing Francis's coefficient value somewhat arbitrarily by 1 per cent. He also made a few experiments, but stated that his formula was subject to the limitations imposed by Francis; consequently the extension of the range of application of the formula has been an excursion into unexplored territory. The notch designed by Cipolletti was intended to measure a minimum discharge of 150 liters (5.3 cubic feet) per second and a maximum discharge of 300 liters (10.6 cubic feet) per second, thus further restricting the use of the Cipolletti formula to notches having crest lengths of not less than 3 feet nor more than 8 feet.

There is great practical need in irrigation practice for weirs with small notches and for measurements with small depths of water over the crests of the notches. It also is important to know that the discharge formulas are correct, as many other forms of measuring devices are commonly calibrated by being hitched in tandem with the weir. For these reasons it was deemed advisable to conduct a series of experiments with notches having thin edges and full contractions (1) to determine whether the Francis and Cipolletti formulas hold for notches of the sizes ordinarily used in irrigation practice and (2), in case the old formulas did not hold, to derive new formulas.

LABORATORY EQUIPMENT AND METHODS

The hydraulic laboratory at Fort Collins was built in 1912–13, under a cooperative agreement between the Office of Experiment Stations, United States Department of Agriculture, and the Colorado Agricultural Experiment Station, and is designed for research work in hydraulics, especially gravity flow.[1] With the exception of the building, which is of brick, the laboratory is constructed almost entirely of concrete and metal to give it rigidity, permanency, and water-tightness. All water faces of concrete are covered with a 3 to 1 cement-plaster coat three-eighths of an inch thick. Tests have shown the seepage losses to be negligible. The plan and a sectional elevation of the laboratory are shown in figure 1. The circular storage reservoir has a top diameter of 87 feet, side slopes of 1 to 1, and is 6½ feet deep. The headrace connecting it with the weir box is approximately 60 feet long, 4 feet deep, and 6 feet wide for the first 15 feet below the head gates and then expands to 6 feet deep and 10 feet wide at the weir box. The weir box is 20 feet long, 10 feet wide, and 6 feet deep, and has a heavy T-iron frame approximately 3 feet high and 6 feet long in its bulkhead wall. This frame is surfaced, bored for ⅜-inch bolts, and so arranged that the plates containing or forming the notches or orifices and other measuring devices requiring a vertical position can be adjusted accurately for experiments. The joints between the plates and the frame are made

[1] For a complete description of the hydraulic laboratory, see an earlier article by the writer (4).

water-tight by flat rubber gaskets. The water passing through the notches or orifices falls into a concrete spill box 4 feet deep, 10 feet wide, and 9 feet long, which is connected with an auxiliary or waste reser-

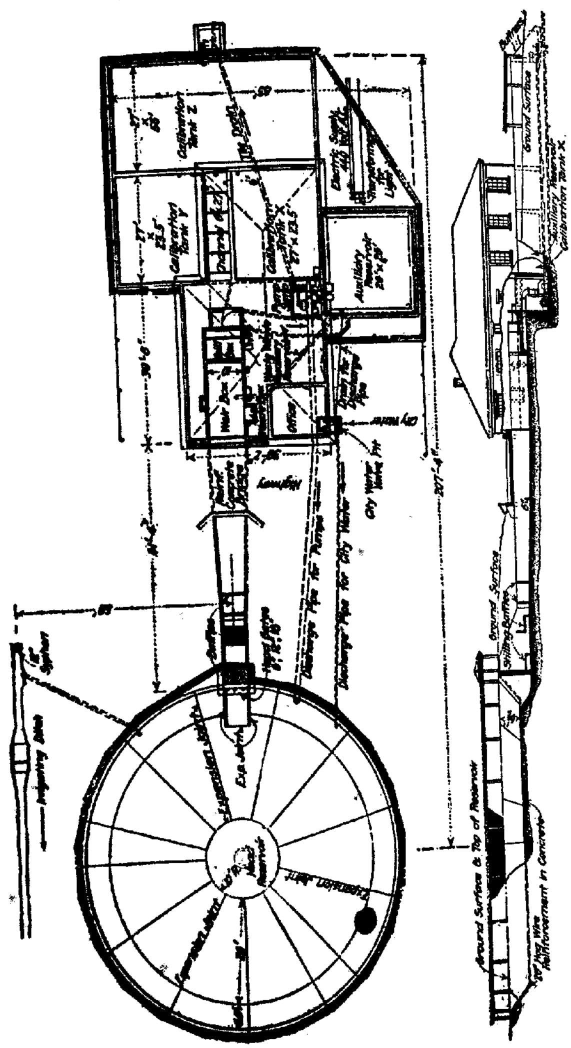

FIG. 1.—Plan and sectional elevations of the Fort Collins hydraulic laboratory.

voir by one channel and with the calibrated tanks by another. The two 22-inch circular openings leading to these channels are separated by a steel plate, and a single disk on the lever arm makes a double shear

gate for the openings. The calibrated tanks and the wasteways on the weir box as well as the spill box are connected with the waste reservoir, from which the water can be returned to the storage reservoir by either a 12-inch or a 5-inch horizontal centrifugal pump driven by electricity. The floors of the calibrated tanks and the waste reservoir are 19 feet lower than the coping of the storage reservoir.

Some of the means used to secure accuracy in the experiments are as follows: The laboratory is so arranged that the centers of the storage reservoir, the headrace, the frame in the end of the weir box, and the channel from the spill box to the calibrated tanks all lie in the same straight line, thus permitting the water to approach and leave the device under experiment in a straight line.

The three head gates between the storage reservoir and the headrace—6, 12, and 18 inches in diameter, respectively—permit a fairly accurate regulation of the water entering the weir box.

Immediately below the head gates a series of two horizontal and two vertical baffles breaks up the eddy currents and reduces pulsations and wave action to such an extent that the water, before entering the weir box, is in a pondlike condition.

In one side of the weir box, about 15 feet upstream from the bulkhead, is an overpour spillway which resembles a door 2 feet high and 3 feet long hinged at the bottom. The top of this spillway when in an upright position is slightly below the top of the weir box. Aprons of oiled canvas attached to the sides of the weir box and to the face of the door prevent leakage and compel the water to pass over the crest of the spillway. A 4-inch gate valve placed at the side of the spillway permits a still more careful regulation of the depth of the water in the weir box. Both the spillway and the gate valve can be adjusted by the hook-gauge observer on the opposite side of the weir box by means of screw controls operated by handwheels placed on the ends of long rods. By always having some water running over the spillway it was possible to keep the head upon the device under test constant throughout the duration of the experiment, usually from 20 to 40 minutes, depending upon the volume of water being run.

The elevations of the water in the weir box and the spill box are observed in concrete gauge boxes built on the outside walls of the respective boxes. These gauge boxes are 1 foot by 2 feet by 4 feet deep, inside dimensions, and the water enters each of them through four 1-inch pipes. The gauge box for the weir box is located 10 feet upstream, and that for the spill box 7 feet downstream from the bulkhead. The pipes leading to the latter, however, take water from the spill box at a point only 3½ feet downstream from the plane of the weir. Each gauge box is equipped with an electric drop light and a Boyden hook gauge anchored in the concrete wall, and readings of the water level can be made to 0.001 of a foot.

In order to refer the elevation of the crest of the notch being experimented with to a reading of the weir-box hook gauge to the nearest 0.001 foot, the instrument shown in figure 2 was devised. The ends of the legs and the hook can be adjusted so as to make the distance from the top of the plate to the groove in the legs exactly equal to the distance from the top of the plate to the point of the hook. By resting the notched legs on the crest of the notch and adjusting the plate to a horizontal position with a sensitive level, the point of the hook is brought to the same elevation as the crest of the notch. Water is run into the weir box, and the surface of the water is adjusted to the point of the crest-hook gauge. Since it is possible to maintain the water level in the weir box quite accurately, the hook-gauge reading in the weir-box gauge box is taken to correspond to the crest elevation of the notch. Repeated determinations of this nature indicated a high degree of accuracy.

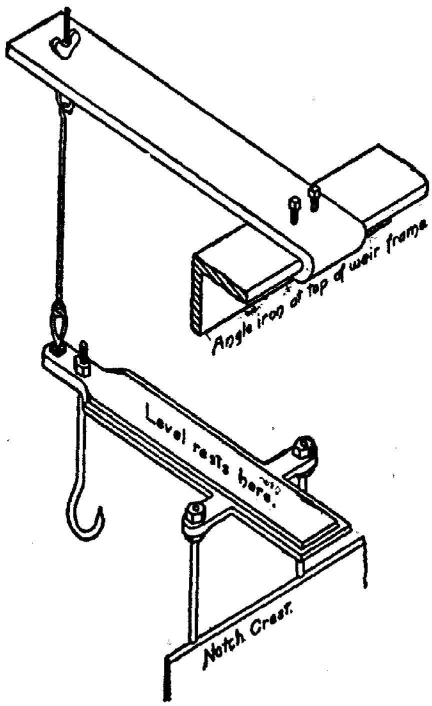

FIG. 2.—Device used in referring elevations of the notch crest to the reading of the hook gauge.

In order to avoid the fluctuating conditions of the flow which occur when tests are being started or stopped, means had to be provided for quickly turning the flow into the channel to the calibrated tanks when the desired conditions for the test had been obtained. This is accomplished by means of the double shear gate used to close the two 22-inch circular openings in the spill box. The lever arm of this gate is 8 feet long, the disk is seated by means of steel shear springs, and the gate is positive and instantaneous in action. When the gate handle reaches midpoint of its swing, it strikes a gong, which is a signal to the hook-gauge observer to start or stop the stop watch used in recording the

duration of the experiments. The error in time in operating the shear gate and the stop watch is only a small fraction of a second.

The calibrated tanks cover an area 55 feet square, divided by 12-inch vertical-sided concrete walls into one tank 27 by 55 feet, two tanks each 23½ by 27 feet, and a channel 6 by 27 feet, which is connected with each tank by a 14-inch circular orifice placed on the floor line and controlled by a gate. The tanks are 8½ feet deep. Their floors are all at the same elevation, and they have a combined capacity of more than 22,000 cubic feet available for experimental purposes. The tanks have been carefully calibrated, corrections having been made for all irregularities, gate openings, rods, etc., and tables have been prepared giving the capacity at each 0.001 foot in elevation. A brass rod 1 inch in diameter and 9 feet long was placed in a vertical position near one corner in each calibrated tank, being held out from the wall about 6 inches by iron brackets set in the concrete (fig. 3). Holes drilled in these rods at carefully measured intervals of about 18 inches serve as datum points when the quantity of water in the tanks is being measured. The elevation of the water in the tanks is determined to 0.001 foot by means of a hook gauge having fixed to its back a heavy clamp provided with a pin which fits snugly into the holes in the rod. A steel ladder was placed adjacent to the brass standard rods in each tank and anchored to the concrete. The platform shown in figure 3 is 20 by 24 inches and can be lowered close to the water surface and secured to any of the ladders by means of hooks. The funnel-shaped arrangement attached to the platform has a ½-inch hole in the bottom and can be adjusted so as to form a stilling basin for the hook gauge. With the water levels at the beginning and the end of the experiment determined by means of the standard rod and hook gauge, the volume run during the experiment can be determined readily from the calibration tables.

Unless otherwise stated, the experiments recorded in this publication were made with notches the edges of which were one-sixteenth inch or less in thickness. The notch plates used were constructed either entirely of brass or of steel with brass notch edges. The crests and sides of the notches were dressed to true angles and straight lines, and by means of a micrometer caliper were calibrated to an allowable divergence of 0.002 inch from a straight line. The triangular notches were dressed to templates. The plate containing the notch under observation was placed in a vertical position in the T frame in the bulkhead of the weir box, and the crests of rectangular and Cipolletti notches were leveled to within 0.001 foot by means of a 12-inch steel-frame level, upon which a bubble division indicated a variation of 0.0004 foot for a length of 1 foot. The inner face of the bulkhead was flush with the crest of the notch. The triangular notch plates were placed so that a vertical line would bisect the angle formed by the sides of the notch. In all the experi-

ments except those upon the effect of contraction (p. 1091) the bottom of the weir box was approximately 4½ feet below the crests of the notches, and the sides of the weir box were 3 to 4½ feet from the ends of the crests, depending upon the size of the notches. In all the experiments the floor

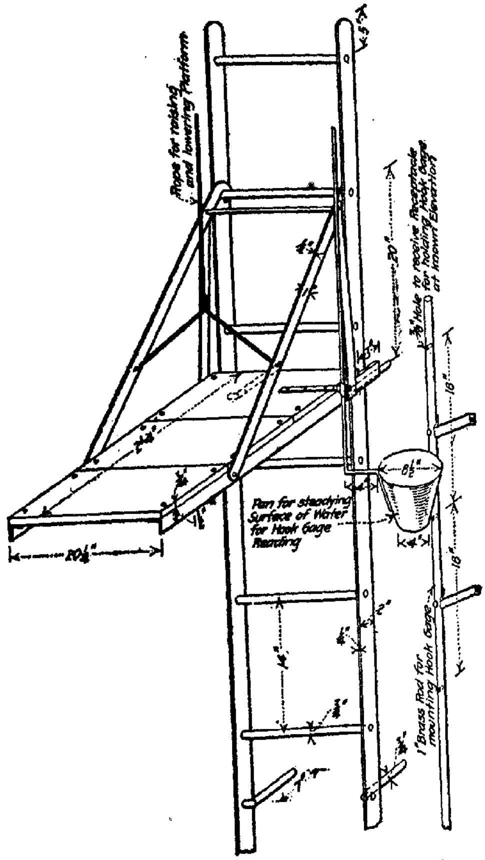

Fig. 5.—Ladder, platform, and datum rod used in calibration tanks.

of the spill box was approximately 4.5 feet below the vertex, or crest, of the notch.

Thirty or forty tests were made upon each notch, the experimental variable being the head. Intervals of head of 0.05 foot were used, and duplicate tests were run for each 0.1 foot of head. If the data from the duplicate tests did not agree within one-half of 1 per cent, the tests were

repeated until such agreement was obtained. It is not claimed that this arbitrary rule insures the accuracy of results of the individual tests, but it did lead to the detection of irregularities in the working conditions and increased the probability of accuracy. Comparatively few tests had to be rerun, which indicates the stability of the experimental tests and the nice control of the heads made possible by the head gates, wasteways, and baffles.

The heads and the corresponding discharges obtained were plotted for the various notches. The curves were then drawn which best represented the discharges through the different notches, the plottings being made upon such a scale that discharge values could be read from the curves to three decimal places.

The following method was used in smoothing the curves and obtaining the values for C in the general formula $Q=CLH^n$:

Discharge values were taken from the curves for each 0.05 foot head, and the slope was determined for each straight line connecting pairs of points. The slope for each point was first taken as the average between the slopes of the two straight lines to which it was common; then, calling the point in question b, the point for the next 0.05 foot head above, a, and that below, c, the slopes were given a second smoothing by the equation $\frac{a+2b+c}{4}=b$; and a third smoothing was obtained by substituting the values obtained by the second smoothing in the equation $\frac{a+2b+3c+2d+e}{9}=c$. These values were plotted, and the equation of the resulting curve was used to compute the last smoothing of the slopes. Substituting these computed values for n in the general formula $Q=CLH^n$, the corresponding value of C was obtained for each head.

EXPERIMENTS WITH NOTCHES HAVING FREE FLOW

DEDUCTIONS OF FORMULAS FOR RECTANGULAR AND TRAPEZOIDAL NOTCHES

The general type of formula heretofore used for discharges through rectangular and trapezoidal notches is $Q=CLH^n$, in which L is length of crest, H the head of water over the crest, and C and n are constant for each type of weir. Expressed logarithmically, the general formula becomes $\log Q=\log C+\log L+n \log H$, which equation, when plotted, gives a straight line whose slope is n and whose intercept is $\log C+\log L$.

The data obtained for the rectangular and Cipolletti notches, when plotted logarithmically, gave curves instead of straight lines. It was found, however, that a general straight-line equation could be deduced for the discharges through the rectangular notches, which, within the range of the experiments, would give discharges as close to the experi-

mental data as would the general curve equation. The experimental data indicate, however, that the general curve equation would hold true for a greater range of notch lengths and heads than would the general straight-line equation. Table I, for the Cipolletti notches, gives the discharge values for the different heads as read from the curve, the experimental discharge values (observed discharges) at greatest variance with the curve discharge values, and the values of the exponents *n* and coefficients *C* necessary in the Cipolletti formula to give the discharges obtained in the experiments. The values of *n* and *C* in the table show that the discharges for any notch, if plotted logarithmically, would not give a straight line, since neither the *n*'s nor the *C*'s are constant. A comparison of the curve discharge values and the observed discharges in the table also serves to indicate the accuracy of the experimental data. The variations of the *n*'s and *C*'s also hold for the rectangular notches, but are not so pronounced as in the case of the Cipolletti notches, since the discharge curves for rectangular notches are flatter.

TABLE I.—*Discharges through Cipolletti notches, and the exponents and coefficients necessary in using the Cipolletti formula*

Head.	0.50062-foot notch. Discharge, cubic feet per second.		(n)	(C)	1.0050-foot notch. Discharge, cubic feet per second.		(n)	(C)	1.5028-foot notch. Discharge, cubic feet per second.		(n)	(C)
	Observed.	Curve.			Observed.	Curve.			Observed.	Curve.		
Feet.												
0.20	0.152	0.149	1.530	3.492	0.300	0.300	1.498	3.327	0.459	0.455	1.486	3.309
.30	.287	.284	1.565	3.733	.554	.554	1.517	3.424	.829	.829	1.499	3.353
.40	.453	.452	1.600	3.911	.866	.865	1.536	3.516	1.289	1.282	1.513	3.413
.50	.653	.655	1.636	4.066	1.216	1.217	1.555	3.558	1.801	1.798	1.526	3.446
.60	.890	.890	1.671	4.174	1.626	1.622	1.574	3.608	2.378	2.375	1.539	3.469
.70	1.158	1.159	1.706	4.254	2.079	2.072	1.593	3.639	3.018	3.013	1.553	3.488
.80	1.455	1.455	1.742	4.284	2.565	2.562	1.612	3.653	3.718	3.708	1.566	3.500
.90	1.784	1.786	1.778	4.303	3.109	3.111	1.631	3.676	4.458	4.463	1.579	3.508
1.00	2.149	2.155	......	......	3.693	3.696	1.650	3.678	5.262	5.262	1.592	3.502
1.10	......	......	......	......	4.332	4.340	......	......	6.138	6.134	1.605	3.503
1.20	......	......	......	......	......	......	......	......	7.062	7.060	1.618	3.498
1.30	......	......	......	......	......	......	......	......	8.022	8.035	1.632	3.484

Head.	2.0002-foot notch. Discharge, cubic feet per second.		(n)	(C)	3.0011-foot notch. Discharge, cubic feet per second.		(n)	(C)	4.0086-foot notch. Discharge, cubic feet per second.		(n)	(C)
	Observed.	Curve.			Observed.	Curve.			Observed.	Curve.		
Feet.												
0.20	0.603	0.602	1.480	3.255	0.909	0.902	1.473	3.217	1.200	1.206	1.470	3.205
.30	1.100	1.100	1.490	3.307	1.633	1.648	1.480	3.263	2.200	2.191	1.476	3.232
.40	1.693	1.695	1.501	3.353	2.538	2.538	1.487	3.303	3.342	3.365	1.481	3.261
.50	2.323	2.373	1.511	3.381	3.528	3.528	1.495	3.314	4.700	4.700	1.487	3.287
.60	3.146	3.142	1.521	3.416	4.663	4.634	1.502	3.326	6.172	6.180	1.493	3.305
.70	3.952	3.952	1.532	3.412	5.890	5.870	1.509	3.350	7.807	7.795	1.499	3.319
.80	4.836	4.845	1.542	3.417	7.173	7.190	1.516	3.360	9.532	9.532	1.504	3.326
.90	5.815	5.814	1.553	3.424	8.574	8.574	1.523	3.354	11.342	11.390	1.510	3.331
1.00	6.792	6.844	1.563	3.422	10.081	10.080	1.530	3.359	13.376	13.376	1.515	3.337
1.10	7.943	7.941	1.573	3.417	11.655	11.655	1.538	3.354	15.467	15.425	1.521	3.329
1.20	9.116	9.107	1.584	3.411	13.347	13.355	1.545	3.357	......	......	......	......
1.30	......	......	......	......	......	......	......	......	......	......	......	......

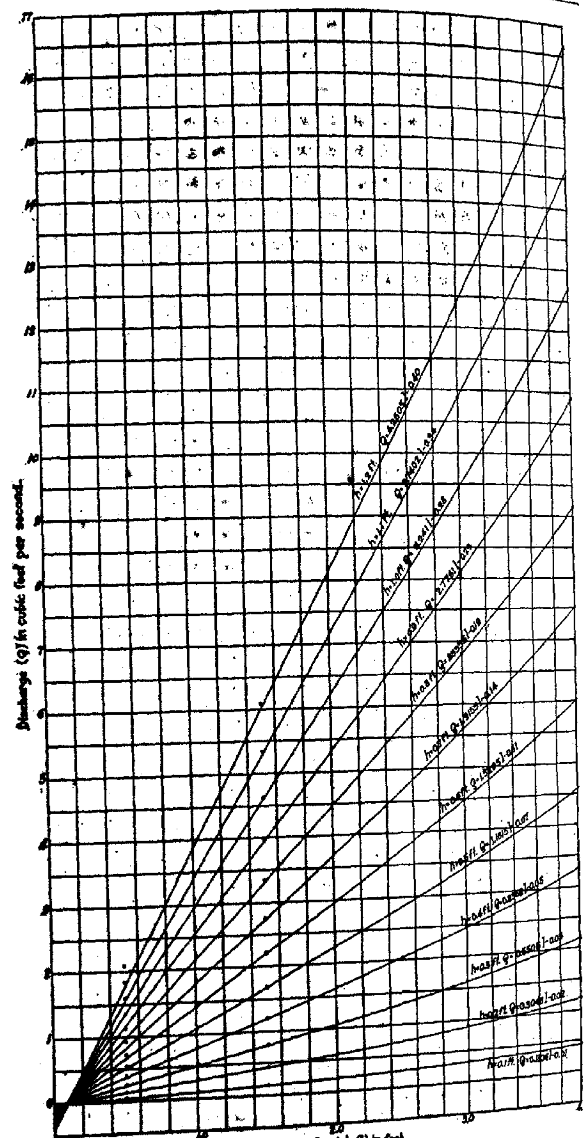

FIG. 4.—Curves showing the relation between discharges with constant heads through rectangular notches of different lengths and the lengths of the notches.

RECTANGULAR NOTCHES

With rectangular notches 226 tests were made, the actual crest lengths used being 0.50721 foot, 1.0055 feet, 1.5026 feet, 2.0057 feet, 2.9970 feet, and 4.0065 feet. These actual lengths were used in all computations connected with the derivation of the formula.

DERIVATION OF THE FORMULA

The discharge values for 0.05-foot increments of head, taken from the curves plotted from the experimental data, were used in the following deductions, thereby eliminating to a large extent the experimental

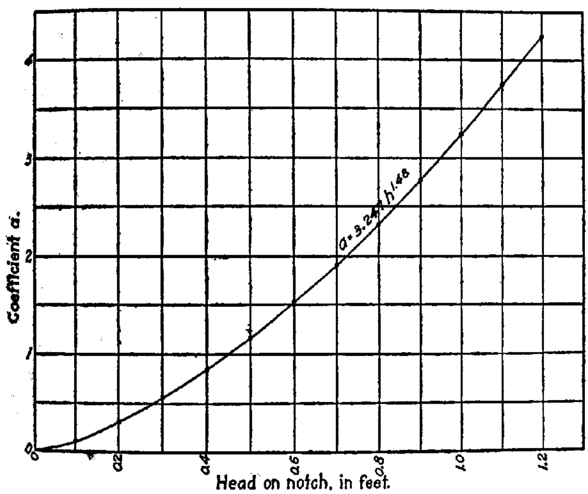

FIG. 5.—Curve showing the relation between a in the equation $Q=aL-b$ and the heads on rectangular notches.

irregularities. The discharge values for the different notches were plotted (fig. 4) with the lengths of crests (L) as abscissas, and the discharges (Q) as ordinates. A straight line was then drawn for each head by passing it through the points representing the discharges over the 3- and 4-foot crests with the given head. The equations of these straight lines were found to be of the form $Q=aL-b$.

The slopes (a) of the lines were computed from the coordinates of the discharge values with the 3- and 4-foot crests. The relations between the heads (H) and the slopes (a) in the above formula were plotted (fig. 5) and gave a curve the equation for which was found to be $a=3.247H^{1.48}$.

The relations between the heads (H) and the intercepts (b) in the equation $Q=aL-b$ are shown in figure 6. The equation for the curve was found to be $b=0.283H^{1.9}$.

The offsets from each of the straight lines in figure 4 to the points representing the discharges with the head for which the line was drawn were tabulated, and an expression for the offsets was determined to be $\frac{0.283H^{1.9}}{1+2L^{1.8}}$.

Substituting the values of a and b in the equation form $Q=aL-b$ and making a correction for the offsets from the straight lines, the formula for the rectangular notches was found to be

$$Q=3.247\ LH^{1.48}-\left(\frac{0.566L^{1.8}}{1+2L^{1.8}}\right)H^{1.9}$$

Table II gives the discharge values for the rectangular notches of different lengths computed by this formula. This formula gives discharge

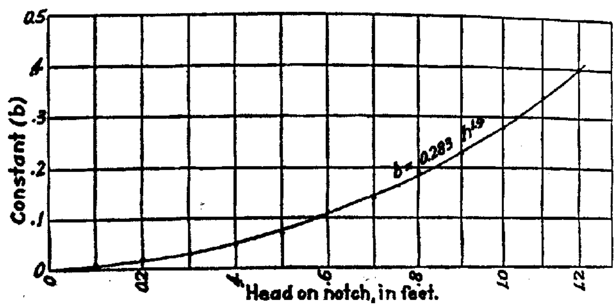

FIG. 6.—Curve showing the relation between b in the equation $Q=aL-b$ and the heads on rectangular notches.

values within a maximum of approximately 1.2 per cent of the values indicated on the curves plotted from the experimental data, but the average variation is only 0.28 per cent. Table V compares the values indicated on the curves plotted from the experimental data and values computed with formulas.

TABLE II.—*Discharges (in cubic feet per second) through rectangular weir notches*[1]

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
0.20	2⅜	0.291	0.439	0.588	0.887	1.19
.21	2½	.312	.472	.632	.954	1.28
.22	2⅝	.335	.505	.677	1.02	1.37
.23	2¾	.358	.539	.723	1.09	1.46
.24	2⅞	.380	.574	.769	1.16	1.55

[1] Computed by the formula $Q=3.247LH^{1.48}-\left(\frac{0.566L^{1.8}}{1+2L^{1.8}}\right)H^{1.9}$

TABLE II.—*Discharges (in cubic feet per second) through rectangular weir notches*—Con.

Head. Feet.	Head. Inches.	1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
0.25	3	0.404	0.609	0.817	1.23	1.65
.26	3⅛	.428	.646	.865	1.31	1.75
.27	3¼	.452	.682	.914	1.38	1.85
.28	3⅜	.477	.720	.965	1.46	1.95
.29	3½	.502	.758	1.02	1.53	2.05
.30	3⅝	.527	.796	1.07	1.61	2.16
.31	3¾	.553	.836	1.12	1.69	2.27
.32	3 13/16	.580	.876	1.18	1.77	2.37
.33	3 15/16	.606	.916	1.23	1.86	2.48
.34	4 1/16	.634	.957	1.28	1.94	2.60
.35	4 3/16	.661	.999	1.34	2.02	2.71
.36	4 5/16	.688	1.04	1.40	2.11	2.82
.37	4 7/16	.717	1.08	1.45	2.20	2.94
.38	4 9/16	.745	1.13	1.51	2.28	3.06
.39	4 11/16	.774	1.17	1.57	2.37	3.18
.40	4 13/16	.804	1.21	1.63	2.46	3.30
.41	4 15/16	.833	1.26	1.69	2.55	3.42
.42	5 1/16	.863	1.30	1.75	2.65	3.54
.43	5 3/16	.893	1.35	1.81	2.74	3.67
.44	5¼	.924	1.40	1.88	2.83	3.80
.45	5⅜	.955	1.44	1.94	2.93	3.93
.46	5½	.986	1.49	2.00	3.03	4.05
.47	5⅝	1.02	1.54	2.07	3.12	4.18
.48	5¾	1.05	1.59	2.13	3.22	4.32
.49	5⅞	1.08	1.64	2.20	3.32	4.45
.50	6	1.11	1.68	2.26	3.42	4.58
.51	6⅛	1.15	1.73	2.33	3.52	4.72
.52	6¼	1.18	1.78	2.40	3.62	4.86
.53	6⅜	1.21	1.84	2.46	3.73	4.99
.54	6½	1.25	1.89	2.53	3.83	5.13
.55	6⅝	1.28	1.94	2.60	3.94	5.27
.56	6¾	1.31	1.99	2.67	4.04	5.42
.57	6 13/16	1.35	2.04	2.74	4.15	5.56
.58	6 15/16	1.38	2.09	2.81	4.26	5.70
.59	7 1/16	1.42	2.15	2.88	4.36	5.85
.60	7 3/16	1.45	2.20	2.96	4.47	6.00
.61	7 5/16	1.49	2.25	3.03	4.58	6.14
.62	7 7/16	1.52	2.31	3.10	4.69	6.29
.63	7 9/16	1.56	2.36	3.17	4.81	6.44
.64	7 11/16	1.60	2.42	3.25	4.92	6.59
.65	7 13/16	1.63	2.47	3.33	5.03	6.75
.66	7 15/16	1.67	2.53	3.40	5.15	6.90
.67	8 1/16	1.71	2.59	3.48	5.26	7.05
.68	8 3/16	1.74	2.64	3.56	5.38	7.21
.69	8¼	1.78	2.70	3.63	5.49	7.36
.70	8⅜	1.82	2.76	3.71	5.61	7.52
.71	8½	1.86	2.81	3.78	5.73	7.68
.72	8⅝	1.90	2.87	3.86	5.85	7.84
.73	8¾	1.93	2.93	3.94	5.97	8.00
.74	8⅞	1.97	2.99	4.02	6.09	8.17

TABLE II.—*Discharges (in cubic feet per second) through rectangular weir notches*—Con.

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
0.75	9	2.01	3.05	4.10	6.21	8.33
.76	9⅛	2.05	3.11	4.18	6.33	8.49
.77	9¼	2.09	3.17	4.26	6.45	8.66
.78	9⅜	2.13	3.23	4.34	6.58	8.82
.79	9½	2.17	3.29	4.42	6.70	8.99
.80	9⅝	2.21	3.35	4.51	6.83	9.16
.81	9¾	2.25	3.41	4.59	6.95	9.33
.82	9 13/16	2.29	3.47	4.67	7.08	9.50
.83	9 15/16	2.33	3.54	4.75	7.21	9.67
.84	10 1/16	2.37	3.60	4.84	7.33	9.84
.85	10 3/16	2.41	3.66	4.92	7.46	10.01
.86	10 5/16	2.46	3.72	5.01	7.59	10.19
.87	10 7/16	2.50	3.79	5.10	7.72	10.36
.88	10 9/16	2.54	3.85	5.18	7.85	10.54
.89	10 11/16	2.58	3.92	5.27	7.99	10.71
.90	10 13/16	2.62	3.98	5.35	8.12	10.89
.91	10 15/16	2.67	4.05	5.44	8.25	11.07
.92	11 1/16	2.71	4.11	5.53	8.38	11.25
.93	11 3/16	2.75	4.18	5.62	8.52	11.43
.94	11¼	2.79	4.24	5.71	8.65	11.61
.95	11⅜	2.84	4.31	5.80	8.79	11.79
.96	11½	2.88	4.37	5.89	8.93	11.98
.97	11⅝	2.93	4.44	5.98	9.06	12.16
.98	11¾	2.97	4.51	6.07	9.20	12.34
.99	11⅞	3.01	4.57	6.15	9.34	12.53
1.00	12	3.06	4.64	6.25	9.48	12.72
1.01	12⅛	....	4.71	6.34	9.62	12.91
1.02	12¼	....	4.78	6.43	9.76	13.10
1.03	12⅜	....	4.85	6.52	9.90	13.28
1.04	12½	....	4.92	6.62	10.04	13.47
1.05	12⅝	....	4.98	6.71	10.18	13.66
1.06	12¾	....	5.05	6.80	10.32	13.85
1.07	12 13/16	....	5.12	6.90	10.46	14.04
1.08	12 15/16	....	5.19	6.99	10.61	14.24
1.09	13 1/16	....	5.26	7.09	10.75	14.43
1.10	13 3/16	....	5.34	7.19	10.90	14.64
1.11	13 5/16	....	5.41	7.28	11.04	14.83
1.12	13 7/16	....	5.48	7.38	11.19	15.03
1.13	13 9/16	....	5.55	7.47	11.34	15.22
1.14	13 11/16	....	5.62	7.57	11.49	15.42
1.15	13 13/16	....	5.69	7.66	11.64	15.62
1.16	13 15/16	....	5.77	7.76	11.79	15.82
1.17	14 1/16	....	5.84	7.86	11.94	16.02
1.18	14 3/16	....	5.91	7.96	12.09	16.23
1.19	14¼	....	5.98	8.06	12.24	16.43
1.20	14⅜	....	6.06	8.16	12.39	16.63
1.21	14½	....	6.13	8.26	12.54	16.83
1.22	14⅝	....	6.20	8.36	12.69	17.04
1.23	14¾	....	6.28	8.46	12.85	17.25
1.24	14⅞	....	6.35	8.56	12.99	17.45

TABLE II.—*Discharges (in cubic feet per second) through rectangular weir notches*—Con.

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
1.25	15	....	6.43	8.66	13.14	17.66
1.26	15⅛	....	....	....	13.30	17.87
1.27	15¼	....	....	....	13.45	18.07
1.28	15⅜	....	....	....	13.61	18.28
1.29	15½	....	....	....	13.77	18.50
1.30	15⅝	....	....	....	13.93	18.71
1.31	15¾	....	....	....	14.09	18.92
1.32	15$\frac{13}{16}$	....	....	....	14.24	19.13
1.33	15$\frac{15}{16}$	....	....	....	14.40	19.34
1.34	16$\frac{1}{16}$	....	....	....	14.56	19.55
1.35	16$\frac{3}{16}$	....	....	....	14.72	19.77
1.36	16$\frac{5}{16}$	....	....	....	14.88	19.98
1.37	16$\frac{7}{16}$	....	....	....	15.04	20.20
1.38	16$\frac{9}{16}$	....	....	....	15.20	20.42
1.39	16$\frac{11}{16}$	....	....	....	15.36	20.64
1.40	16$\frac{13}{16}$	....	....	....	15.53	20.86
1.41	16$\frac{15}{16}$	....	....	....	15.69	21.08
1.42	17$\frac{1}{16}$	....	....	....	15.85	21.30
1.43	17$\frac{3}{16}$	....	....	....	16.02	21.52
1.44	17¼	....	....	....	16.19	21.74
1.45	17⅜	....	....	....	16.34	21.96
1.46	17½	....	....	....	16.51	22.18
1.47	17⅝	....	....	....	16.68	22.41
1.48	17¾	....	....	....	16.85	22.63
1.49	17⅞	....	....	....	17.01	22.85
1.50	18	....	....	....	17.18	23.08

The discharges through a notch having a crest length of 0.5 foot did not follow the same law as those through larger notches. This was probably owing to the greater effect of friction in the smaller notch and to the interference due to the end-contraction filaments of flow crossing each other in the middle of the notch section. The formula

$$Q = 1.593H^{1.526}\left(1 + \frac{1}{800H^{2.3}}\right)$$

was found to give discharge values consistent with the curve plotted from experimental data for the 0.5-foot notch. The use of such a notch is very limited, and the 90° triangular notch is as accurate and much more satisfactory.

Comparison of the Francis Formula and the New Formula

The discharge values obtained for rectangular notches by the Francis and the new formulas are shown in graphic form in figure 7 and in tabular form in Table III.

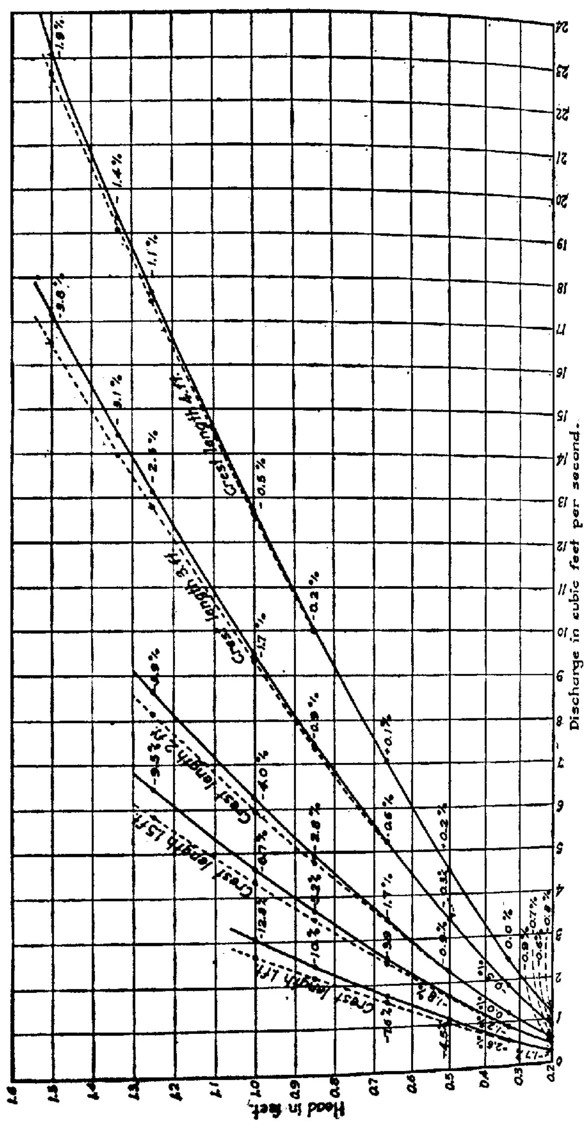

FIG. 7.—Curves showing discharges through rectangular notches of different lengths.

TABLE III.—*Comparison of discharges through rectangular notches computed from the Francis formula and the new formula*

Head.	1-foot crest.			1½-foot crest.			2-foot crest.			3-foot crest.			4-foot crest.		
	Discharge computed by new formula (cubic feet per second).	Discharge computed by the Francis formula.[1]		Discharge computed by new formula (cubic feet per second).	Discharge computed by the Francis formula.[1]		Discharge computed by new formula (cubic feet per second).	Discharge computed by the Francis formula.[1]		Discharge computed by new formula (cubic feet per second).	Discharge computed by the Francis formula.[1]		Discharge computed by new formula (cubic feet per second).	Discharge computed by the Francis formula.[1]	
		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.
Feet.															
0.20	0.291	0.286	98.3	0.439	0.435	99.1	0.588	0.584	99.3	0.887	0.882	99.4	1.19	1.18	99.2
.33	.606	.590	97.4	.916	.905	98.8	1.22	1.22	100.0	1.86	1.85	99.5	2.48	2.48	100.0
.50	1.11	1.06	95.5	1.68	1.65	98.2	2.26	2.24	99.1	3.42	3.41	99.7	4.58	4.59	100.2
.67	1.71	1.58	92.4	2.59	2.49	96.1	3.47	3.41	98.3	5.26	5.23	99.4	7.05	7.06	100.1
.85	2.41	2.17	90.0	3.66	3.47	94.8	4.92	4.78	97.2	7.45	7.38	99.1	10.01	9.99	99.8
1.00	3.06	2.67	87.2	4.64	4.33	93.3	6.24	5.99	96.0	9.48	9.32	98.3	12.72	12.65	99.5
1.25	....	....	....	6.43	5.82	90.5	8.65	8.14	94.1	13.14	12.80	97.4	17.65	17.45	98.9
1.33	....	....	....	....	....	....	....	....	....	14.40	13.96	96.9	19.34	19.07	98.6
1.50	....	....	....	....	....	....	....	....	....	17.17	16.52	96.2	23.08	22.64	98.1

[1] The Francis formula: $Q=3.33(L-.2H)H^{3/2}$.

The curves and Table III show that except for a small range of heads on the 4-foot notch the discharges computed by the Francis formula are too small. The actual discharges, however, where the head did not exceed one-third of the length of the crest, did not vary much from those computed by the Francis formula and support the statement of Francis that his formula would give discharge values correct to within 2 per cent, provided the head does not exceed one-third the length of the crest. Nevertheless the fact that the curves plotted from the experimental data have no sudden breaks or changes of direction shows that no limit need be placed upon the head, provided the proper formula is used to compute the discharge. It also shows that the necessity of the limit on the application of the Francis formula was due to the mathematical shortcoming of the formula and not to any peculiarity inherent in the rectangular notch. The new formula not only gives greater accuracy within the range of the Francis formula but also permits the accurate measurement of discharges with the heads exceeding one-third the length of the crest. The maximum limit of the ratio of the head to the crest length with the new formula has not been ascertained, the greatest ratio experimented with being 1 to 1 with the 1-foot notch. The parts of all the curves showing the discharges with higher heads, however, were quite consistent in all cases with the rest of the curves. A head of 1 foot was run over a 0.5-foot notch, but the results were inconclusive, as the discharges through the 0.5-foot notch do not follow the general formula.

The new formula is more complicated than the Francis formula, but gives discharge values which are more accurate within the limits of these experiments, and since tables are generally consulted to determine the flow that is passing through a notch, the practical disadvantage of the new formula is largely overcome. If one is obliged to use a formula in the field for computing the discharge, an approximation usually is sufficient, and the Francis formula gives discharges sufficiently accurate for practical needs.

Straight-line Formula

As stated on page 1059, it was found, when the experimental data for the rectangular notches were plotted logarithmically, that a general straight-line formula could be deduced which, within the range of the experiments, would give discharge values as close to the plotted values as did the general formula deduced above. The equations for the straight lines best representing the discharges with the given heads through the different notches were found to be as shown in Table IV.

TABLE IV.—*Equations for straight lines representing discharges through rectangular weir notches*

Length of crest.	Equations of line.
Feet.	
1.0055	$Q=3.078LH^{1.463}$
1.5026	$Q=3.106LH^{1.465}$
2.0057	$Q=3.125LH^{1.466}$
2.9970	$Q=3.154LH^{1.467}$
4.0056	$Q=3.172LH^{1.473}$

The coefficient values (C) in the above equations were plotted (fig. 8) against the lengths of crests (L), and the exponent values

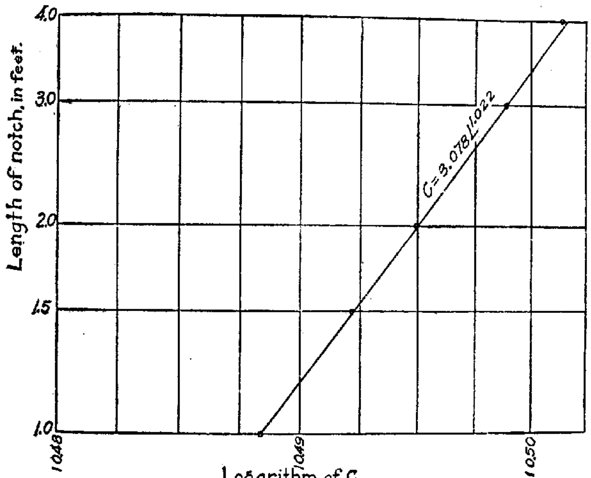

FIG. 8.—Curve showing relation of coefficients (C) to lengths of rectangular notches.

(n) were plotted (fig. 9) against the lengths of crests (L). Average straight lines drawn to represent the points were found to have the equations $C=3.078L^{1.022}$ and $n=1.46+0.003L$.

Substituting these values of C and n in the equation $Q=CLH^n$, the formula for the discharge through rectangular notches was found to be

$$Q=3.08L^{1.022}H^{(1.46+.003L)}.$$

This formula gives discharge values that agree within a maximum of 0.7 per cent with the values indicated on the curves plotted from the experimental data, but the average variation is only 0.26 per cent.

Table V gives the discharges through the notches used, computed by the curve and by the straight-line formulas, also the values indicated on the curves plotted from the experimental data.

TABLE V.—*Discharges (in cubic feet per second) for rectangular notches as shown by curves plotted from experimental data, and discharges computed by curve and straight-line formulas*

Head.	1.0055-foot notch. Experimental data.	1.0055-foot notch. Curve formula.	1.0055-foot notch. Straight-line formula.	1.5026-foot notch. Experimental data.	1.5026-foot notch. Curve formula.	1.5026-foot notch. Straight-line formula.	2.0057-foot notch. Experimental data.	2.0057-foot notch. Curve formula.	2.0057-foot notch. Straight-line formula.	2.997-foot notch. Experimental data.	2.997-foot notch. Curve formula.	2.997-foot notch. Straight-line formula.	4.0056-foot notch. Experimental data.	4.0056-foot notch. Curve formula.	4.0056-foot notch. Straight-line formula.
Feet.															
0.2	0.293	0.293	0.294	0.443	0.440	0.442	0.593	0.590	0.593	0.890	0.886	0.889	1.194	1.189	1.190
.3	.531	.530	.532	.800	.797	.801	1.079	1.071	1.074	1.617	1.610	1.613	2.163	2.161	2.161
.4	.806	.808	.811	1.220	1.217	1.220	1.640	1.635	1.637	2.461	2.462	2.461	3.302	3.304	3.302
.5	1.115	1.120	1.123	1.680	1.688	1.692	2.267	2.268	2.271	3.411	3.418	3.416	4.594	4.589	4.585
.6	1.459	1.462	1.467	2.195	2.205	2.210	2.969	2.964	2.966	4.474	4.470	4.465	6.013	6.004	5.997
.7	1.834	1.830	1.838	2.755	2.761	2.770	3.718	3.716	3.719	5.595	5.605	5.600	7.532	7.533	7.524
.8	2.233	2.223	2.235	3.354	3.357	3.368	4.519	4.519	4.523	6.795	6.821	6.814	9.157	9.171	9.156
.9	2.660	2.639	2.655	3.988	3.987	4.002	5.367	5.369	5.375	8.090	8.110	8.101	10.910	10.906	10.891
1.0	3.103	3.076	3.097	4.664	4.650	4.670	6.238	6.265	6.273	9.432	9.467	9.457	12.706	12.734	12.720
1.1	......	......	......	5.370	5.346	5.369	7.190	7.205	7.214	10.866	10.893	10.878	14.642	14.656	14.635
1.2	......	......	......	6.133	6.068	6.099	8.174	8.181	8.195	12.356	12.374	12.361	16.666	16.653	16.635
1.3	......	......	......	6.903	6.819	6.857	9.193	9.196	9.215	13.876	13.918	13.903	......	......	......

In locating the straight lines on the logarithmic plot, it was found that the points for the 1.0055-foot notch could be covered quite closely by three straight lines approximately equal in length. The same was approximately true of the points for the 1.5026-foot notch. Only two

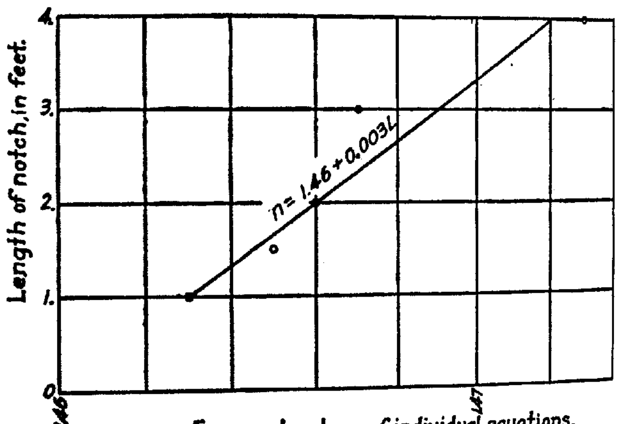

FIG. 9.—Curve showing relation of *n* to length of rectangular notches.

straight lines each, however, were required for the 2.0057-foot and 2.997-foot notches, although a third could be assumed near the upper part of the curves in each case. For the 4.0056-foot notch there was only one point of change, and it was well above the middle of the curve. These facts indicate that had large enough heads been run on the longer notches to give the same ratio of length of crest to head as was obtained with the

1-foot notch, an equal number of lines would have been required to cover the points. If a single straight line is taken to represent the discharge curve, and it is placed to represent best the discharges with the lower heads, as was done above, the part of the true discharge curve for the higher heads diverges rapidly from the straight line. The curve formula takes account of the law of variation of the discharge curves better than does the straight-line formula, and, consequently, it appears that it will give closer values for the higher heads and for longer notches than those experimented with.

The straight-line equation for the 0.5-foot notch was found to be $Q=1.566H^{1.604}$.

This equation was found to give discharge values within approximately 1 per cent of the values indicated on the curve plotted from the experimental data.

CIPOLLETTI NOTCHES

With notches having side slopes of one horizontal to four vertical, 219 tests were made. The actual crest lengths used were 0.50062 foot, 1.0050 feet, 1.5028 feet, 2.0002 feet, 3.0011 feet, and 4.0058 feet, respectively, and these lengths were used throughout the following calculations.

Derivation of the Formula

The difference between the areas of a Cipolletti and a rectangular notch with equal crest length is the area of a 28° 4′ (approximately) triangular notch—that is, one having one to four side slopes. It was found, however, that the discharges through such a notch (see Table X) with a given head did not exactly equal the difference between the discharges through a rectangular and a Cipolletti notch with equal crest lengths and the same head. While the differences between the discharges through the Cipolletti and rectangular notches increase with the head for all crest lengths, there was no regular increase or decrease in the differences in the discharges with increases in the crest lengths so long as the heads were less than approximately 0.8 foot, but for higher heads the differences in discharges decreased as the crest lengths increased. The comparison of the differences is very unreliable for heads as low as 0.2 or 0.3 foot. The discharges through the 28° 4′ notch are greater than the differences between the discharges of the Cipolletti and rectangular notches for all heads up to approximately 2.5 feet, the percentages of excess decreasing with the increases in head and equaling zero with a head of approximately 2.5.

The differences between the discharges through the rectangular and Cipolletti notches for each of the crest lengths were determined from the curves plotted from the experimental data and an average made for each 0.1 foot of head. These averages were then plotted logarithmically against the head, and the equation of the curve representing the differ-

ence in discharge was found to be $D = .609H^{2.5}$. By adding the term $.609H^{2.5}$ to the general formula for discharges through rectangular notches (page 1064), the general formula for discharges through Cipolletti notches was found to be

$$Q = 3.247LH^{1.48} - \left(\frac{0.566L^{1.8}}{1+2L^{1.8}}\right)H^{1.9} + 0.609H^{2.5}$$

This formula gives discharge values for 1-, 1½-, 2-, 3-, and 4-foot notches that agree within 0.5 per cent of the values indicated on the curves plotted from the experimental data, except for the lower heads on the 1-foot notch, where the maximum discrepancy, owing to the small discharge, is approximately 1½ per cent. The discrepancies are positive in some cases and negative in others. (See Table VII for discharge values indicated by the curves plotted from the experimental data and discharge values computed by the formulas.)

Table VI gives the discharge values for Cipolletti notches of different lengths computed by the new formula.

TABLE VI.—*Discharges (in cubic feet per second) through Cipolletti weir notches*[1]

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
0.20	2⅜	0.30	0.45	0.60	0.90	1.20
.21	2½	.32	.48	.64	.97	1.29
.22	2⅝	.35	.52	.69	1.04	1.38
.23	2¾	.37	.55	.74	1.11	1.47
.24	2⅞	.39	.59	.79	1.18	1.57
.25	3	.42	.63	.84	1.25	1.67
.26	3⅛	.45	.67	.89	1.33	1.77
.27	3¼	.47	.71	.94	1.40	1.87
.28	3⅜	.50	.75	.99	1.48	1.97
.29	3½	.53	.79	1.04	1.56	2.08
.30	3⅝	.56	.83	1.10	1.64	2.19
.31	3¾	.59	.87	1.15	1.73	2.30
.32	3 13/16	.61	.91	1.21	1.81	2.41
.33	3 15/16	.64	.95	1.27	1.89	2.52
.34	4 1/16	.67	1.00	1.32	1.98	2.64
.35	4 3/16	.70	1.04	1.38	2.07	2.75
.36	4 5/16	.73	1.09	1.44	2.16	2.87
.37	4 7/16	.77	1.13	1.50	2.25	2.99
.38	4 9/16	.80	1.18	1.57	2.34	3.11
.39	4 11/16	.83	1.23	1.63	2.43	3.24
.40	4 13/16	.87	1.28	1.69	2.53	3.36
.41	4 15/16	.90	1.32	1.76	2.62	3.49
.42	5 1/16	.93	1.37	1.82	2.72	3.61
.43	5 3/16	.97	1.42	1.89	2.81	3.74
.44	5¼	1.00	1.47	1.95	2.91	3.87

[1] Computed by the formula $Q = 3.247\, LH^{1.48} - \left(\frac{0.566L^{1.8}}{1+2L^{1.8}}\right)H^{1.9} + 0.609H^{2.5}$

TABLE VI.—*Discharges (in cubic feet per second) through Cipolletti weir notches*—Con.

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
0.45	5⅜	1.04	1.53	2.02	3.01	4.01
.46	5½	1.07	1.58	2.09	3.11	4.14
.47	5⅝	1.11	1.63	2.16	3.21	4.28
.48	5¾	1.15	1.68	2.23	3.32	4.41
.49	5⅞	1.18	1.74	2.30	3.42	4.55
.50	6	1.22	1.79	2.37	3.53	4.69
.51	6⅛	1.26	1.85	2.44	3.64	4.83
.52	6¼	1.30	1.90	2.51	3.74	4.97
.53	6⅜	1.34	1.96	2.59	3.85	5.12
.54	6½	1.38	2.02	2.66	3.96	5.26
.55	6⅝	1.42	2.07	2.74	4.07	5.41
.56	6¾	1.46	2.13	2.81	4.18	5.56
.57	6 13/16	1.50	2.19	2.89	4.30	5.71
.58	6 15/16	1.54	2.25	2.97	4.41	5.86
.59	7 1/16	1.58	2.31	3.05	4.53	6.01
.60	7 3/16	1.62	2.37	3.13	4.64	6.17
.61	7 5/16	1.67	2.43	3.20	4.76	6.32
.62	7 7/16	1.71	2.49	3.28	4.88	6.47
.63	7 9/16	1.75	2.55	3.37	5.00	6.63
.64	7 11/16	1.80	2.62	3.45	5.12	6.79
.65	7 13/16	1.84	2.68	3.53	5.24	6.95
.66	7 15/16	1.89	2.75	3.61	5.36	7.11
.67	8 1/16	1.93	2.81	3.70	5.48	7.28
.68	8 3/16	1.98	2.87	3.79	5.61	7.44
.69	8¼	2.02	2.94	3.87	5.73	7.61
.70	8⅜	2.07	3.01	3.95	5.86	7.77
.71	8½	2.12	3.07	4.04	5.98	7.94
.72	8⅝	2.16	3.14	4.13	6.11	8.11
.73	8¾	2.21	3.21	4.22	6.24	8.28
.74	8⅞	2.26	3.28	4.31	6.38	8.45
.75	9	2.31	3.35	4.40	6.51	8.62
.76	9⅛	2.36	3.42	4.49	6.64	8.80
.77	9¼	2.41	3.49	4.58	6.77	8.97
.78	9⅜	2.46	3.56	4.67	6.90	9.15
.79	9½	2.51	3.63	4.76	7.04	9.33
.80	9⅝	2.56	3.70	4.85	7.18	9.51
.81	9¾	2.61	3.77	4.95	7.31	9.69
.82	9 13/16	2.66	3.84	5.04	7.45	9.87
.83	9 15/16	2.71	3.92	5.14	7.59	10.05
.84	10 1/16	2.77	3.99	5.23	7.73	10.23
.85	10 3/16	2.82	4.07	5.33	7.87	10.42
.86	10 5/16	2.87	4.14	5.43	8.01	10.60
.87	10 7/16	2.93	4.22	5.52	8.15	10.79
.88	10 9/16	2.98	4.29	5.62	8.30	10.98
.89	10 11/16	3.04	4.37	5.72	8.44	11.17

TABLE VI.—*Discharges (in cubic feet per second) through Cipolletti weir notches*—Con.

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
0.90	10 13/16	3.09	4.45	5.82	8.59	11.36
.91	10 15/16	3.15	4.53	5.92	8.73	11.55
.92	11 1/16	3.20	4.60	6.02	8.88	11.74
.93	11 3/16	3.26	4.68	6.13	9.03	11.94
.94	11¼	3.32	4.76	6.23	9.17	12.13
.95	11⅜	3.37	4.84	6.33	9.32	12.33
.96	11½	3.43	4.92	6.44	9.47	12.53
.97	11⅝	3.49	5.00	6.55	9.62	12.72
.98	11¾	3.55	5.09	6.64	9.78	12.92
.99	11⅞	3.61	5.17	6.75	9.93	13.12
1.00	12	3.67	5.25	6.86	10.08	13.32
1.01	12⅛	....	5.33	6.96	10.24	13.53
1.02	12¼	....	5.42	7.07	10.40	13.73
1.03	12⅜	....	5.50	7.18	10.55	13.94
1.04	12½	....	5.59	7.29	10.71	14.15
1.05	12⅝	....	5.67	7.40	10.87	14.35
1.06	12¾	....	5.76	7.51	11.03	14.56
1.07	12 13/16	....	5.84	7.62	11.19	14.77
1.08	12 15/16	....	5.93	7.73	11.35	14.98
1.09	13 1/16	....	6.02	7.84	11.51	15.19
1.10	13 3/16	....	6.11	7.96	11.68	15.41
1.11	13 5/16	....	6.20	8.07	11.84	15.62
1.12	13 7/16	....	6.29	8.18	12.00	15.83
1.13	13 9/16	....	6.38	8.29	12.16	16.04
1.14	13 11/16	....	6.47	8.41	12.33	16.26
1.15	13 13/16	....	6.56	8.53	12.50	16.48
1.16	13 15/16	....	6.65	8.65	12.67	16.70
1.17	14 1/16	....	6.74	8.76	12.84	16.93
1.18	14 3/16	....	6.83	8.88	13.01	17.15
1.19	14¼	....	6.93	9.00	13.18	17.37
1.20	14⅜	....	7.02	9.12	13.35	17.59
1.21	14½	....	7.11	9.24	13.52	17.81
1.22	14⅝	....	7.20	9.36	13.69	18.03
1.23	14¾	....	7.30	9.48	13.87	18.26
1.24	14⅞	....	7.40	9.60	14.04	18.49
1.25	15	....	7.49	9.72	14.21	18.71
1.26	15⅛	....	....	....	14.39	18.94
1.27	15¼	....	....	....	14.56	19.17
1.28	15⅜	....	....	....	14.74	19.41
1.29	15½	....	....	....	14.92	19.65
1.30	15⅝	....	....	....	15.11	19.88
1.31	15¾	....	....	....	15.29	20.12
1.32	15 13/16	....	....	....	15.46	20.35
1.33	15 15/16	....	....	....	15.64	20.58
1.34	16 1/16	....	....	....	15.82	20.82

TABLE VI.—*Discharges (in cubic feet per second) through Cipolletti weir notches*—Con.

Head.		1-foot crest.	1½-foot crest.	2-foot crest.	3-foot crest.	4-foot crest.
Feet.	*Inches.*					
1.35	16 3/16	....	....	....	16.01	21.06
1.36	16 5/16	....	....	....	16.19	21.29
1.37	16 7/16	....	....	....	16.37	21.53
1.38	16 9/16	....	....	....	16.56	21.78
1.39	16 11/16	....	....	....	16.75	22.02
1.40	16 13/16	....	....	....	16.94	22.27
1.41	16 15/16	....	....	....	17.13	22.51
1.42	17 1/16	....	....	....	17.32	22.75
1.43	17 3/16	....	....	....	17.51	23.00
1.44	17¼	....	....	....	17.70	23.25
1.45	17⅜	....	....	....	17.89	23.50
1.46	17½	....	....	....	18.08	23.75
1.47	17⅝	....	....	....	18.28	24.00
1.48	17¾	....	....	....	18.47	24.25
1.49	17⅞	....	....	....	18.66	24.50
1.50	18	....	....	....	18.85	24.75

The discharges through the Cipolletti notch, having a nominal crest length of 0.5 foot, did not follow the same law as those through the longer notches, possibly for the reasons noted on page 1067 for the 0.5-foot rectangular notch, and the use of such notches should be discouraged in favor of the 90° triangular notch, which measures small discharges more accurately.

The following formula represents the flow through the 0.5-foot Cipolletti notch, but is stated here only for technical reasons:

$$Q=1.593H^{1.526}\left(1+\frac{1}{800H^{2.3}}\right)0.587H^{2.53}$$

COMPARISON OF THE CIPOLLETTI FORMULA AND THE NEW FORMULA

The discharge values computed by the Cipolletti and new formulas are shown in graphic form in figure 10 and in tabular form in Table VII.

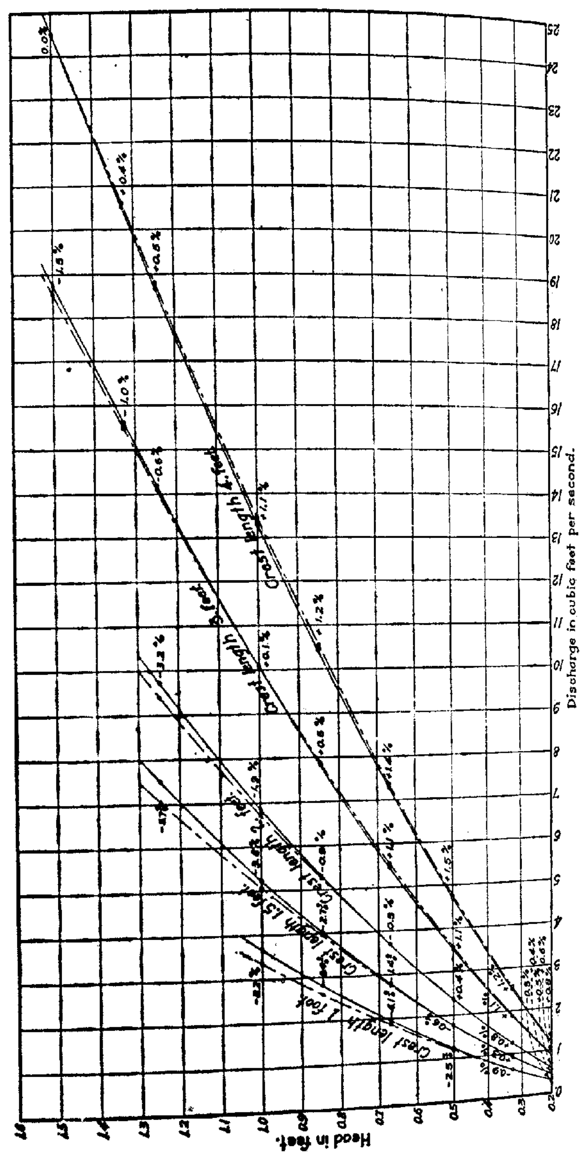

FIG. 10.—Curves showing discharges through Cipolletti weir notches of different lengths.

TABLE VII.—*Comparison of discharges through trapezoidal notches with side slopes of 1:4 computed by the Cipolletti formula and by the new formula.*[1]

Head.	1-foot crest.			1½-foot crest.			2-foot crest.			3-foot crest.			4-foot crest.		
	Discharge computed by new formula (cubic feet per second).	Discharge computed by Cipolletti formula.		Discharge computed by new formula (cubic feet per second).	Discharge computed by Cipolletti formula.		Discharge computed by new formula (cubic feet per second).	Discharge computed by Cipolletti formula.		Discharge computed by new formula (cubic feet per second).	Discharge computed by Cipolletti formula.		Discharge computed by new formula (cubic feet per second).	Discharge computed by Cipolletti formula.	
		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.		Amount (cubic feet per second).	Percentage of discharge computed by new formula.
Feet.															
0.20	0.302	0.301	99.7	0.450	0.452	100.4	0.599	0.602	100.5	0.898	0.903	100.6	1.20	1.21	100.8
.33	.644	.638	99.1	.954	.957	100.3	1.27	1.28	100.8	1.89	1.91	101.1	2.52	2.55	101.2
.50	1.22	1.19	97.5	1.79	1.78	99.4	2.37	2.38	100.4	3.53	3.57	101.1	4.69	4.76	101.5
.67	1.93	1.85	95.9	2.81	2.77	98.6	3.70	3.69	99.7	5.48	5.54	101.1	7.28	7.38	101.4
.85	2.82	2.64	93.6	4.07	3.96	97.3	5.33	5.28	99.1	7.87	7.91	100.5	10.42	10.55	101.2
1.00	3.67	3.37	91.8	5.25	5.05	96.2	6.86	6.73	98.1	10.09	10.10	100.1	13.33	13.47	101.1
1.25	....	....	....	7.49	7.06	94.3	9.72	9.41	96.8	14.21	14.12	99.4	18.72	18.82	100.5
1.33	....	....	....	....	....	....	....	....	....	15.64	15.49	99.0	20.58	20.66	100.4
1.50	....	....	....	....	....	....	....	....	....	18.85	18.56	98.5	24.75	24.74	100.0

[1] Cipolletti formula: $Q=3.367LH^{3/2}$

The curves and the table show that with heads less than one-third the length of the crest the Cipolletti formula gives discharge values within 1.5 per cent of the actual discharges, therefore being somewhat more accurate than the Francis formula. The new formula, like the new formula for the rectangular weir, is not only more nearly accurate than the old formula, but also permits the use of heads greater than one-third the crest length. The maximum limit of the ratio of the head to the crest length was not ascertained, but the parts of the curves for the higher heads are consistent, there being no sudden breaks or changes of direction.

The new formula is more complicated than the Cipolletti formula, but because of its greater degree of accuracy it should be used in computing tables. The Cipolletti formula, however, is sufficiently accurate for field computations where only approximate discharge values are required.

Cipolletti notches do not give discharges proportional to the lengths of the crest, as has been commonly claimed, and consequently notches of this type have no advantages over rectangular notches (see p. 1098).

FORMULA BASED ON THE STRAIGHT-LINE FORMULA FOR RECTANGULAR NOTCHES

The difference between the discharges computed by the new rectangular-notch formula and the discharges taken from the curves plotted from the experimental data for the Cipolletti notches were determined for each 0.1 foot of head for the several lengths of notches. These values were then plotted logarithmically against the heads, and the equation of the average straight line representing the difference in discharge was found to be $D=.6H^{2.6}$. By adding the term $0.6H^{2.6}$ to the general formula for discharges through rectangular notches (p. 1071), the general formula for discharges through Cipolletti notches was found to be

$$Q=3.08L^{1.020}H^{(1.46+0.003L)}+0.6H^{2.6}$$

This formula gives discharge values that agree within a maximum of 1 per cent of the values indicated on the curves plotted from the experimental data, but the agreement is within 0.5 per cent for all but a very few points.

Table VIII gives the discharges through the notches used, computed by the two formulas deduced for the Cipolletti notches, and the discharge values indicated on the curves plotted from the experimental data.

TABLE VIII.—*Discharges (in cubic feet per second) for Cipolletti weir notches as shown by curves plotted from experimental data, and discharges computed by formulas on pages 1074 and 1080*

Head.	1.0050-foot notch.			1.5028-foot notch.			2.0002-foot notch.			3.0011-foot notch.			4.0058-foot notch.		
	Experimental data.	According to formula on page 1074.	According to formula on page 1080.	Experimental data.	According to formula on page 1074.	According to formula on page 1080.	Experimental data.	According to formula on page 1074.	According to formula on page 1080.	Experimental data.	According to formula on page 1074.	According to formula on page 1080.	Experimental data.	According to formula on page 1074.	According to formula on page 1080.
Feet.															
0.2	0.300	0.302	0.303	0.455	0.450	0.451	0.600	0.600	0.602	0.902	0.900	0.898	1.206	1.20	1.199
.3	.555	.563	.558	.829	.83	.827	1.109	1.10	1.100	1.647	1.64	1.639	2.193	2.19	2.188
.4	.866	.874	.866	1.280	1.28	1.275	1.694	1.69	1.694	2.535	2.53	2.519	3.360	3.36	3.357
.5	1.218	1.23	1.222	1.798	1.80	1.791	2.375	2.37	2.370	3.530	3.53	3.515	4.705	4.70	4.684
.6	1.622	1.63	1.626	2.370	2.37	2.369	3.141	3.13	3.125	4.650	4.64	4.624	6.179	6.18	5.156
.7	2.075	2.08	2.077	3.004	3.02	3.009	3.953	3.95	3.958	5.870	5.86	5.839	7.800	7.78	7.763
.8	2.565	2.57	2.571	3.706	3.71	3.704	4.845	4.85	4.859	7.185	7.18	7.150	9.537	9.52	9.492
.9	3.111	3.11	3.111	4.462	4.46	4.458	5.815	5.82	5.831	8.576	8.59	8.557	11.392	11.38	11.348
1.0	3.695	3.69	3.697	5.261	5.26	5.270	6.845	6.86	6.873	10.078	10.08	10.057	13.376	13.34	13.320
1.1	......	......	......	6.137	6.12	6.138	7.941	7.96	7.983	11.655	11.68	11.647	15.425	15.43	15.404
1.2	......	......	......	7.060	7.03	7.063	9.110	9.12	9.159	13.359	13.36	13.325	......	......	......

The differences between the discharges through the 0.5-foot Cipolletti notch obtained from the curves plotted from the experimental data and the discharges computed by the formula for the 0.5-foot rectangular notch were determined and plotted logarithmically against the heads. The straight line representing these differences has the equation $D=0.56H^{2.55}$. By adding the term $0.56H^{2.55}$ to the formula for the discharge through the 0.5-foot rectangular notch, the formula for the discharge through a 0.5-foot Cipolletti notch becomes

$$Q=1.566H^{1.504}+0.56H^{2.55}$$

NOTCHES WITH SIDE SLOPES OF 1 TO 3 AND 1 TO 6

Experiments were made with notches having crest lengths of 2 feet and side slopes of 1 to 3 and 1 to 6, respectively. Since notches of only one length were used in each set of experiments, no general equations were deduced for notches of these types. The discharges obtained in the experiments for heads over 0.4 foot are shown graphically in figure 11. Discharges with heads less than 0.4 foot are approximately the same as those given in Tables II and VI.

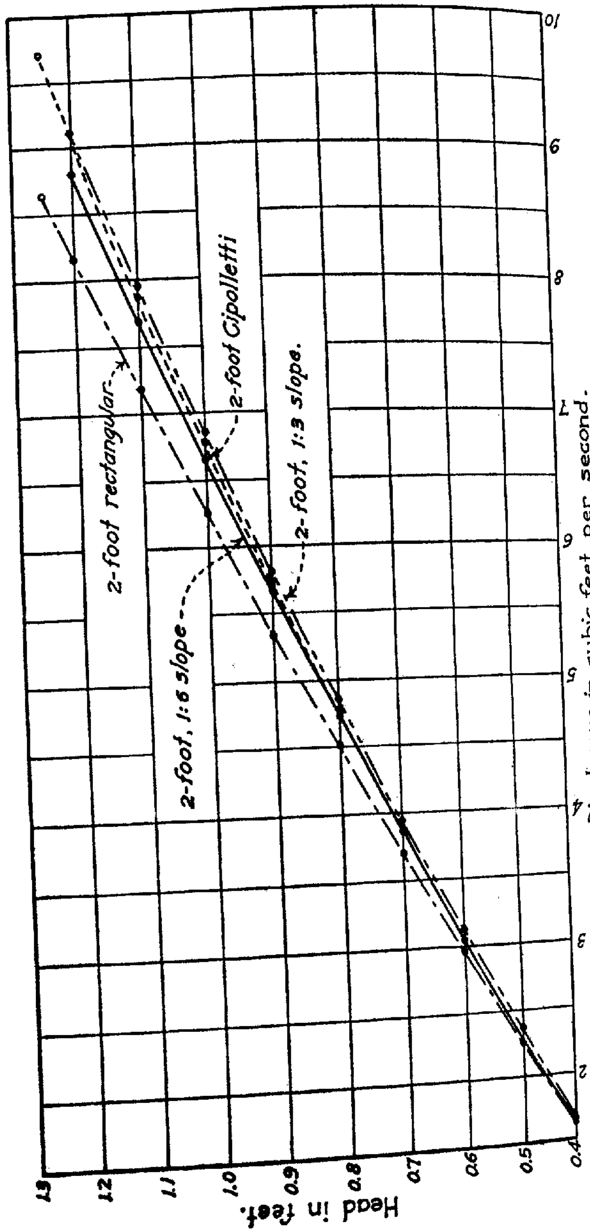

FIG. 11.—Curves showing discharges through 2-foot rectangular and Cipolletti notches and 2-foot notches having 1 to 3 and 1 to 6 side slopes.

TRIANGULAR NOTCHES

General theoretical formulas have been given for triangular notches (7, p. 46; 8, p. 168), and experiments with a 90° notch have been made by Thomson[1] (12, p. 181; 13, p. 154) and Barr.[2] In the Fort Collins laboratory 98 tests were made with heads ranging from 0.2 foot to 1.35 feet on weirs having triangular notches of 120°, 90°, 60°, 30° and approximately 28° 4′. The side slopes for the last-named notch are 1 horizontal to 4 vertical, and the tests were made with the idea that they might be of use in deriving a formula for discharges through Cipolletti notches.

DERIVATION OF FORMULAS

The discharges through the different notches when plotted logarithmically gave straight lines, as shown in figure 12. The equations for these lines were found to be as shown in Table IX.

TABLE IX.—*Equations for straight lines representing discharges through triangular notches*

Notch angle.	Slope of sides, horizontal vertical.	Equation of line.
120°	1. 732	$Q=4.400H^{2.4870}$
90°	1. 000	$Q=2.487H^{2.4805}$
60°	. 577	$Q=1.446H^{2.4705}$
30°	. 268	$Q=0.6848H^{2.4476}$
28°4′ [a]	. 250	$Q=0.6405H^{2.4448}$

[a] Approximate.

The discharging streams had a free fall in all the tests except those for the 120° notch. The upper portion of the stream over the 120° notch adhered to the edge of the notch for a distance of approximately 0.1 foot, the distance being quite uniform for all heads. The sides and crest of the notch used were of brass one-fourth inch thick, and were dressed at an angle of about 45° to a thickness of about one thirty-second inch at the edge. As the amount of adherence of nappe for the 120° notch depends upon the thickness of the edges of the notch, the use of such a notch is impracticable.

The data for the 120° notch having been excluded, the general formula for the discharge through the triangular notches of 28° 4′ to 90° was found to be

$$Q=(0.025+2.462\,S)\,H^{\left(2.5-\frac{0.0195}{S^{0.75}}\right)}$$

[1] The formula derived by Thomson for the 90° notch was $Q=0.305H^{5/2}$, in which Q is in cubic feet per minute and H is in inches.

[2] Barr found that with heads of 2 to 10 inches the coefficient C in Thomson's formula ($Q=CH^{5/2}$) varied from .3104 to .2995. Strickland found that Barr's coefficient C for any head could be computed from the formula $C=0.2907+\frac{0.028}{\sqrt{h}}$, h being in inches.

in which Q is the discharge in cubic feet per second, S is the slope of the sides, expressed decimally, and H is the head in feet.

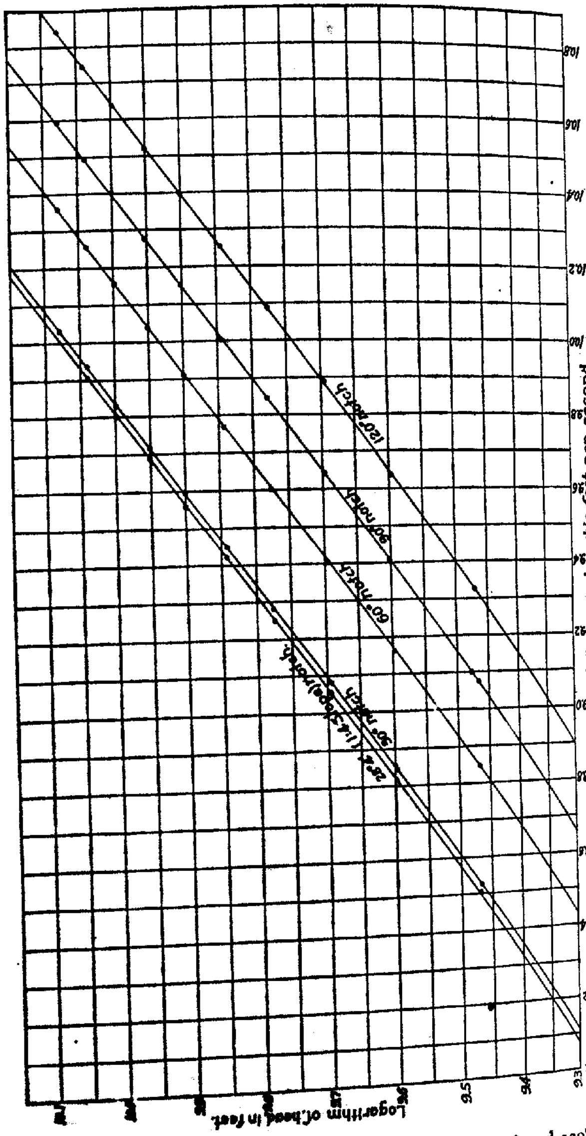

FIG. 12.—Logarithmic diagram of discharges through 28° 4', 30°, 60°, 90°, and 120° triangular notches.

No experiments were made with notches between 90° and 120°, but a ... of the 120° notch led to the conclusion that the

application of the general formula given above can be extended to notches having side slopes of 1 to 1.4 (109° approximately).

Table X, computed by the new general formula, gives the discharges through notches of different shapes with heads up to 1.25 feet.

TABLE X.—*Discharges (in cubic feet per second) for triangular weir notches*[1]

Head.		Notch angle 28° 4'.	Notch angle 30°.	Notch angle 60°.	Notch angle 90°.
Feet.	*Inches.*				
0.20	2 3/8	0.012	0.013	0.027	0.046
.21	2 1/2	.014	.015	.031	.052
.22	2 5/8	.016	.017	.034	.058
.23	2 3/4	.018	.019	.038	.065
.24	2 7/8	.020	.021	.043	.072
.25	3	.022	.023	.047	.080
.26	3 1/8	.024	.025	.052	.088
.27	3 1/4	.026	.028	.057	.096
.28	3 3/8	.029	.030	.062	.105
.29	3 1/2	.031	.033	.068	.115
.30	3 5/8	.034	.036	.074	.125
.31	3 3/4	.037	.039	.080	.136
.32	3 13/16	.040	.042	.087	.147
.33	3 15/16	.043	.045	.094	.159
.34	4 1/16	.046	.049	.101	.171
.35	4 3/16	.049	.052	.108	.184
.36	4 5/16	.053	.056	.116	.197
.37	4 7/16	.056	.060	.124	.211
.38	4 9/16	.060	.064	.132	.225
.39	4 11/16	.064	.068	.141	.240
.40	4 13/16	.068	.073	.150	.256
.41	4 15/16	.072	.077	.160	.272
.42	5 1/16	.077	.082	.170	.289
.43	5 3/16	.081	.087	.180	.306
.44	5 1/4	.086	.092	.190	.324
.45	5 3/8	.091	.097	.201	.343
.46	5 1/2	.096	.102	.212	.362
.47	5 5/8	.101	.108	.224	.382
.48	5 3/4	.106	.114	.236	.403
.49	5 7/8	.112	.120	.248	.424
.50	6	.118	.126	.261	.445
.51	6 1/8	.123	.132	.274	.468
.52	6 1/4	.129	.138	.287	.491
.53	6 3/8	.136	.145	.301	.515
.54	6 1/2	.142	.152	.315	.539
.55	6 5/8	.148	.159	.330	.564
.56	6 3/4	.155	.166	.345	.590
.57	6 13/16	.162	.173	.360	.617
.58	6 15/16	.169	.181	.376	.644
.59	7 1/16	.176	.188	.392	.672

[1] Computed by the formula $Q=(0.025+2.462S)H^{\left(2.5-\frac{0.0195}{S^{0.75}}\right)}$

TABLE X.—*Discharges (in cubic feet per second) for triangular weir notches*—Continued

Head.		Notch angle 28° 4′.	Notch angle 30°.	Notch angle 60°.	Notch angle 90°.
Feet.	*Inches.*				
0.60	7 3/16	0.184	0.196	0.409	0.700
.61	7 5/16	.191	.204	.426	.730
.62	7 7/16	.199	.212	.444	.760
.63	7 9/16	.207	.221	.462	.790
.64	7 11/16	.215	.230	.480	.822
.65	7 13/16	.223	.239	.499	.854
.66	7 15/16	.232	.248	.518	.887
.67	8 1/16	.241	.257	.537	.921
.68	8 3/16	.250	.266	.557	.955
.69	8 1/4	.259	.276	.578	.991
.70	8 3/8	.268	.286	.599	1.03
.71	8 1/2	.277	.296	.620	1.06
.72	8 5/8	.287	.306	.642	1.10
.73	8 3/4	.297	.317	.664	1.14
.74	8 7/8	.307	.328	.687	1.18
.75	9	.317	.339	.710	1.22
.76	9 1/8	.327	.350	.734	1.26
.77	9 1/4	.338	.361	.758	1.30
.78	9 3/8	.349	.373	.782	1.34
.79	9 1/2	.360	.385	.807	1.39
.80	9 5/8	.371	.397	.833	1.43
.81	9 3/4	.383	.409	.859	1.48
.82	9 13/16	.394	.421	.885	1.52
.83	9 15/16	.406	.434	.912	1.57
.84	10 1/16	.418	.447	.940	1.61
.85	10 3/16	.430	.460	.968	1.66
.86	10 5/16	.443	.473	.996	1.71
.87	10 7/16	.456	.487	1.02	1.76
.88	10 9/16	.469	.501	1.05	1.81
.89	10 11/16	.482	.515	1.08	1.86
.90	10 13/16	.495	.529	1.11	1.92
.91	10 15/16	.509	.544	1.15	1.97
.92	11 1/16	.522	.558	1.18	2.02
.93	11 3/16	.536	.573	1.21	2.08
.94	11 1/4	.551	.589	1.24	2.13
.95	11 3/8	.565	.604	1.27	2.19
.96	11 1/2	.580	.620	1.31	2.25
.97	11 5/8	.595	.636	1.34	2.31
.98	11 3/4	.610	.652	1.38	2.37
.99	11 7/8	.625	.668	1.41	2.43
1.00	12	.641	.685	1.45	2.49
1.01	12 1/8	.656	.702	1.48	2.55
1.02	12 1/4	.672	.719	1.52	2.61
1.03	12 3/8	.688	.736	1.56	2.68
1.04	12 1/2	.705	.754	1.59	2.74
1.05	12 5/8	.722	.772	1.63	2.81
1.06	12 3/4	.739	.790	1.67	2.87
1.07	12 13/16	.756	.808	1.71	2.94
1.08	12 15/16	.773	.827	1.75	3.01
1.09	13 1/16	.791	.846	1.79	3.08

TABLE X.—*Discharges (in cubic feet per second) for triangular weir notches*—Con.

Head.		Notch angle 28° 4'.	Notch angle 30°.	Notch angle 60°.	Notch angle 90°.
Feet.	*Inches.*				
1.10	13 3/16	0.809	0.865	1.83	3.15
1.11	13 5/16	.827	.884	1.87	3.22
1.12	13 7/16	.845	.904	1.91	3.30
1.13	13 9/16	.864	.924	1.96	3.37
1.14	13 11/16	.882	.944	2.00	3.44
1.15	13 13/16	.901	.964	2.04	3.52
1.16	13 15/16	.921	.985	2.09	3.59
1.17	14 1/16	.940	1.01	2.13	3.67
1.18	14 3/16	.960	1.03	2.18	3.75
1.19	14 1/4	.980	1.05	2.22	3.83
1.20	14 3/8	1.00	1.07	2.27	3.91
1.21	14 1/2	1.02	1.09	2.32	3.99
1.22	14 5/8	1.04	1.11	2.36	4.07
1.23	14 3/4	1.06	1.14	2.41	4.16
1.24	14 7/8	1.08	1.16	2.46	4.24
1.25	15	1.11	1.19	2.51	4.33

Although weirs with triangular notches are well suited to a comparatively wide range of discharges, they are especially well adapted for the measurement of small discharges and may be used to measure accurately quantities so small that they would not pass through trapezoidal or rectangular notches without adhering to the crests. The use of weirs with triangular notches requires slightly more fall than is required with trapezoidal or rectangular notches—that is, a head of 2 feet is required to deliver approximately 14 cubic feet per second through a 90° triangular notch, while the same discharge would be delivered through a 3-foot rectangular notch with a head of 1.31 feet, or through a 4-foot rectangular notch with a head of 1.07 feet.

Weirs with 90° notches are simpler in construction than any other type of weir and are the most practical type for small or medium-sized discharges. The approximate formula $Q=2.49H^{2.48}$ gives discharge values for 90° notches, which agree very closely with the values obtained with the general formula.

COMPARISON OF NEW FORMULA AND OLD FORMULA

The discharges for the 90° notch computed by the new and the old formulas are compared in Table XI:

TABLE XI.—*Comparison of new formula and old formula*

Head.	Discharge computed by new formula (cubic feet per second).	Discharge computed by old formula, $Q=2.53H^{5/2}$.	
		Discharge in cubic feet per second.	Percentage of discharge computed by new formula.
Feet.			
0.20	0.046	0.045	97.8
.33	.159	.158	99.4
.50	.445	.447	100.4
.67	.921	.930	101.0
.85	1.66	1.69	101.8
1.00	2.49	2.53	101.6
1.25	4.33	4.42	102.1

As no experiments have been made in the past to determine the coefficients in general formulas for notches of 28° 4′, 30°, or 60°, no comparison could be made with the discharges through such notches computed with the new formula.

CIRCULAR NOTCHES

Apparently no experiments have ever been made with circular or semicircular notches placed in a vertical position with heads less than the height of the opening. In order to throw light upon the probable discharges through such notches and obtain data to use in determining the flow through circular head gates when acting as weirs rather than as orifices, 50 tests were made with thin-edged circular notches, 17 being with a notch 0.4995 foot in diameter and 33 with a notch 1.0025 feet in diameter; and 34 tests were made with semicircular notches, 15 being with a notch 1.5011 feet in diameter and 19 with a notch 1.9990 feet in diameter. The discharge data obtained are shown graphically in figure 13.

CONDITIONS OF NOTCH EDGES REQUIRED TO INSURE FREE FLOW

The impression is common that the terms "thin edges" and "sharp crests," as applied to weir notches, mean knife edges. Such edges are not necessary, and the edges are sufficiently sharp or thin if the upstream corner of the notch edges is a distinct angle of 90° or less and the thickness of the notch edges is not so great that the water will adhere to them. The allowable thickness of the edges depends upon the head that is being used. Experiments made in the laboratory with notches having edges

¼ inch thick showed that while water would adhere to the notch edges

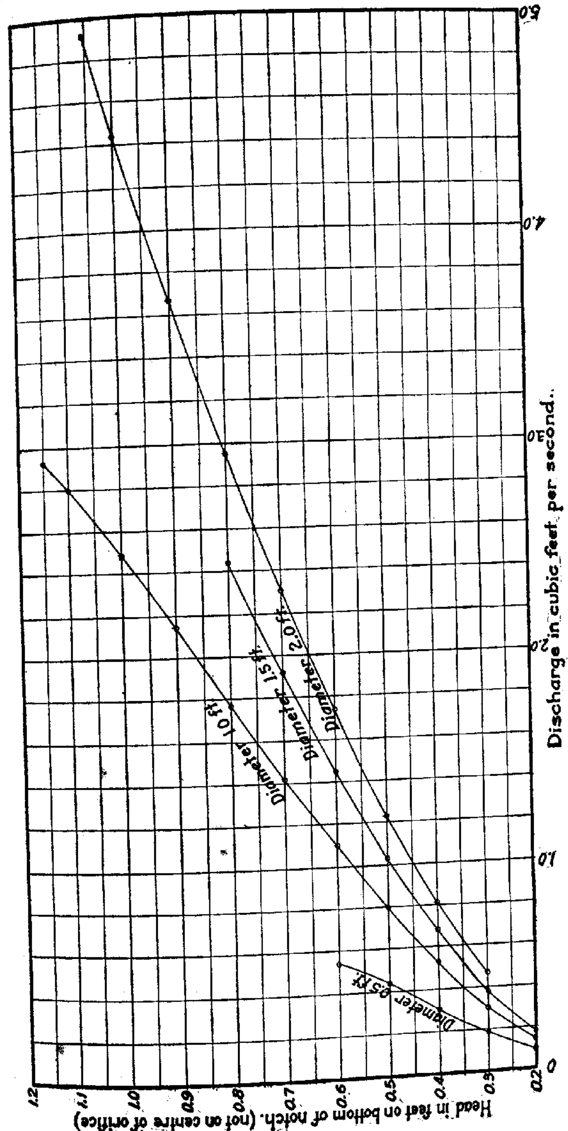

Fig. 13.—Curves showing discharges through circular weir notches.

with a head of 0.15 foot, there was no adherence with heads of 0.2 foot and over.

Notches with angles made as precisely as those used in the test would not be practicable for field use, and consequently a maximum thickness of ⅛ inch probably would be safer than ¼ inch where heads as low as 0.2 foot will be used. While no experiments were made, edges as thick as ¾ inch probably can be used where the minimum head will be 1 foot.

The edges of the weir notches must be straight, true, and rigid. These conditions are best insured by using angle irons or similar material that can be securely fastened to the bulkheads, as wood edges become splintered and warped, and thin sheet-metal weir plates buckle and bend easily. Regardless of the material used, the notches will be more permanent and reliable if the upstream corners of the notches are made definitely angular and the edges are left as thick as possible and still permit a free flow.

DISTANCE FROM NOTCH AT WHICH HEAD SHOULD BE MEASURED

In connection with the experiments with notches of different types, measurements were made to determine the transverse and longitudinal curves of the water surface upstream from the weirs when different heads were being used. These measurements showed that the extent of the curves backward from and to the sides of the notches depends upon the length of the crest and the head being used. Plots of the data obtained show that measurements of head should be made either at a distance of at least $4H$ upstream from the notch or at a distance of at least $2H$ sidewise from the end of the crest of the notch.

Table XII gives the errors and the percentage of error made in computing discharges for notches of different shapes and sizes with different heads caused by errors of 0.01 foot in reading the heads.

TABLE XII.—*Errors and percentage of error in computed discharges caused by 0.01-foot error in reading the heads*

RECTANGULAR WEIRS

Correct head.	Error.									
	1-foot crest.		1½-foot crest.		2-foot crest.		3-foot crest.		4-foot crest.	
Feet.	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*
0.20	0.021	7.22	0.033	7.52	0.044	7.48	0.067	7.55	0.09	7.56
.30	.026	4.94	.04	5.03	.05	4.67	.08	4.97	.10	4.63
.40	.029	3.61	.05	4.13	.06	3.68	.09	3.66	.12	3.64
.50	.04	3.60	.05	2.98	.07	3.10	.10	2.92	.14	3.06
.60	.04	2.76	.05	2.57	.07	2.36	.11	2.46	.14	2.33
.70	.04	2.20	.06	2.17	.07	1.89	.12	2.14	.16	2.13
.80	.04	1.81	.06	1.79	.08	1.77	.12	1.76	.17	1.86
.90	.05	1.92	.07	1.76	.09	1.68	.13	1.60	.18	1.65
1.00	.05	1.63	.07	1.51	.09	1.44	.14	1.48	.19	1.49
1.10	....	....	.07	1.31	.09	1.25	.14	1.28	.19	1.30
1.20	....	....	.07	1.16	.10	1.23	.15	1.21	.20	1.20
1.30	....	....	....	....	....	....	.16	1.15	.21	1.12
1.40	....	....	....	....	....	....	.16	1.03	.22	1.05
1.50	....	....	....	....	....	....	.16	.93	.23	1.00

TABLE XII.—*Errors and percentage of error in computed discharges caused by 0.01-foot error in reading the heads*—Continued

CIPOLLETTI WEIRS

Correct head.	Error.									
	1-foot crest.		1½-foot crest.		2-foot crest.		3-foot crest.		4-foot crest.	
	Cu. ft. per sec.	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*	*Cu. ft. per sec.*	*Per ct.*
Feet. 0.20	0.022	7.3	0.034	7.6	0.045	7.5	0.068	7.6	0.09	7.5
.30	.028	5.0	.041	5.0	.055	5.0	.082	5.0	.11	5.0
.40	.034	3.9	.05	3.9	.07	4.1	.09	3.6	.12	3.6
.50	.04	3.3	.05	2.8	.07	3.0	.11	3.1	.14	3.0
.60	.04	2.5	.06	2.5	.08	2.6	.12	2.6	.15	2.4
.70	.05	2.4	.07	2.3	.09	2.3	.13	2.2	.17	2.2
.80	.05	2.0	.07	1.9	.09	1.9	.14	1.9	.18	1.9
.90	.05	1.6	.08	1.8	.10	1.7	.15	1.7	.19	1.7
1.00	.06	1.6	.08	1.5	.11	1.6	.15	1.5	.20	1.5
1.10	....	....	.09	1.5	.12	1.5	.17	1.5	.21	1.4
1.20	....	....	.09	1.3	.12	1.3	.17	1.3	.22	1.3
1.30	....	....	....	....	....	....	.18	1.2	.24	1.2
1.40	....	....	....	....	....	....	.19	1.1	.24	1.1

90° TRIANGULAR WEIRS

0.20	0.006	13.04	....	....	....	....	....	....	....	....
.50	.022	4.94	....	....	....	....	....	....	....	....
.70	.04	3.9	....	....	....	....	....	....	....	....
1.00	.06	2.4	....	....	....	....	....	....	....	....
1.25	.09	2.1	....	....	....	....	....	....	....	....

EFFECTS OF DIFFERENT END AND BOTTOM CONTRACTIONS UPON DISCHARGES

RECTANGULAR AND CIPOLLETTI NOTCHES

To determine the effect of different end and bottom contractions upon the discharges through rectangular and Cipolletti notches, 120 tests were made with 1-foot rectangular notches, 72 with 3-foot rectangular notches, 205 with 1-foot Cipolletti notches, and 89 with 3-foot Cipolletti notches. Heads of 0.2 foot, 0.6 foot, and 1 foot were used with each notch. The end contractions (the distances of the sides of the weir box from the ends of the crest) and the bottom contraction (the distance of the bottom of the weir box below the crest of the notch) for each notch were varied from 0.5 foot to 3 feet by increments of 0.5 foot. The discharges under the different conditions were compared with those obtained with the standard weir box. The small error in the experimental determinations of the discharges with a 0.2-foot head caused such large percentages of error in the discharges that they were unreliable and so were not included.

Figures 14 and 15 and Tables XIII and XIV show the percentages of increase in discharges and the velocities of approach with heads of 0.6 foot and 1 foot under the different conditions of contractions. The equations of the curve are all of the general form, $e=a(V+b)^n$, in which e

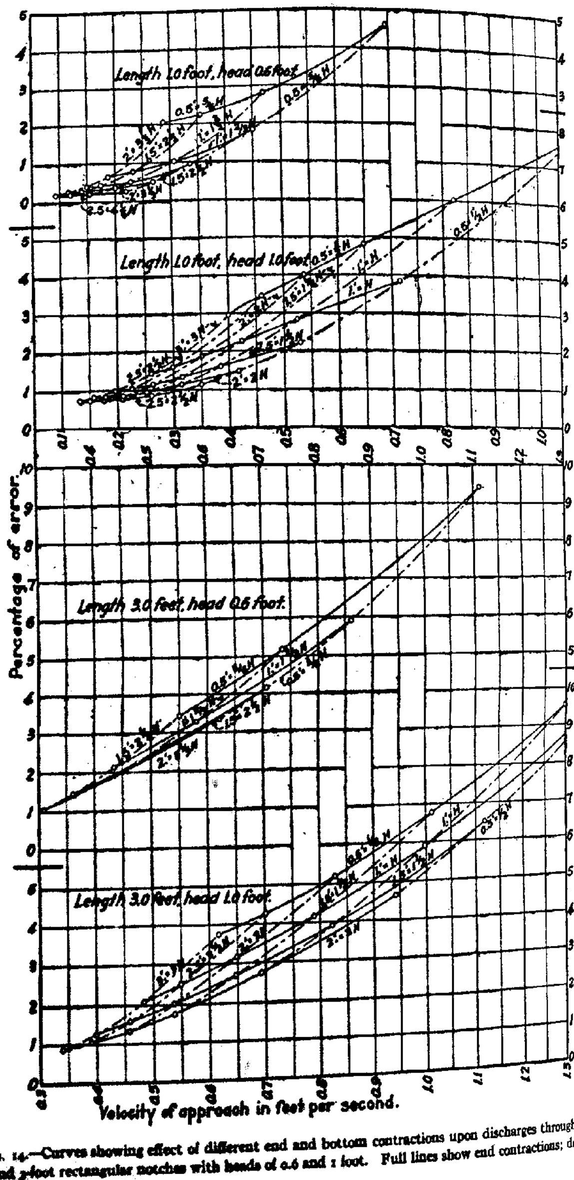

FIG. 14.—Curves showing effect of different end and bottom contractions upon discharges through 1-foot and 3-foot rectangular notches with heads of 0.6 and 1 foot. Full lines show end contractions; dot-dash lines show side contractions.

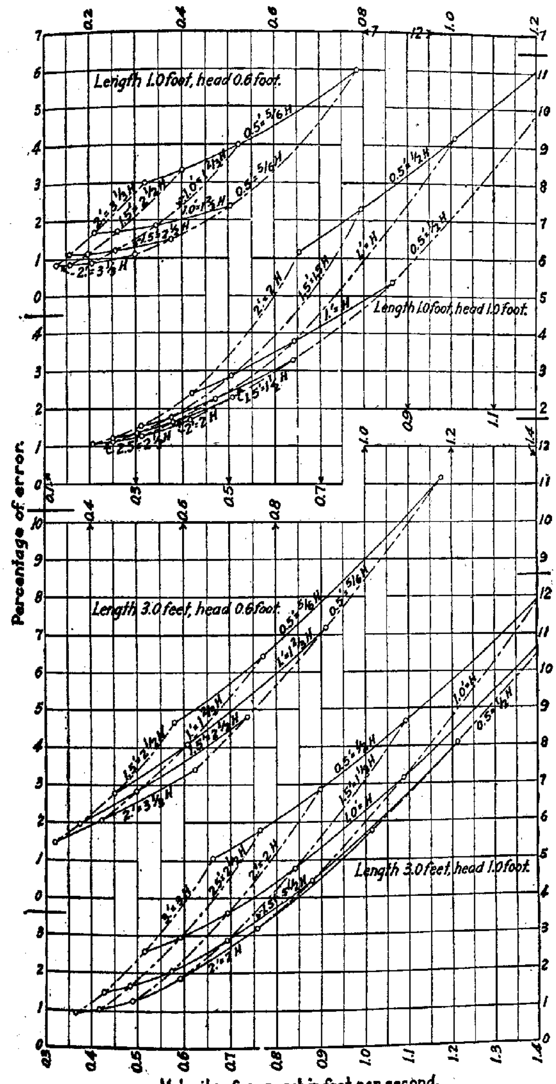

FIG. 15.—Curves showing the effect of different end and bottom contractions upon the discharges through 1-foot and 3-foot Cipolletti weir notches with heads of 0.6 and 1 foot. Full lines show end contractions in feet; dot-dash lines show bottom contractions in feet.

is the percentage of increase in discharge, V is the average velocity of approach, and a, b, and n are constants for each size of each type of notch.

TABLE XIII.—*Velocities of approach (in feet per second) and percentages of increase in discharges through rectangular notches caused by different end and bottom contractions*

HEAD, 0.6 FOOT

Bottom contraction.	End contractions.	1-foot notch. Velocity of approach.	1-foot notch. Increase of discharge.	1½-foot notch. Velocity of approach.	1½-foot notch. Increase of discharge.	2-foot notch. Velocity of approach.	2-foot notch. Increase of discharge.	3-foot notch. Velocity of approach.	3-foot notch. Increase of discharge.	4-foot notch. Velocity of approach.	4-foot notch. Increase of discharge.
Feet.	*Feet.*		*Per ct.*		*Per ct.*		*Per ct.*		*Per ct.*		*Per ct.*
2	2.5	0.094	0.17								
	2.0	.115	.26								
	1.5	.148	.39								
	1.0	.188	.66								
	.5	.288	2.05								
1½	2.5	.119	.17								
	2.0	.141	.30	0.191	0.53	0.239	0.74	0.308	1.07	0.365	1.33
	1.5	.175	.40	.234	.73	.286	1.01	.363	1.44	.416	1.74
	1.0	.234	.76	.304	1.24	.361	1.62	.435	2.12	.489	2.49
	.5	.355	2.26					.552	3.41		
1	2.5	.154	.19								
	2.0	.209	.36								
	1.5	.229	.50	.311	1.09	.377	1.55	.478	2.25	.552	2.79
	1.0	.308	1.01	.400	1.77	.476	2.39	.577	3.22	.650	3.83
	.5	.469	2.84	.573	3.74	.646	4.38	.735	5.15	.794	5.64
½	2.5	.221	.25								
	2.0	.268	.50	.368	1.30	.460	2.05	.624	3.34	.711	4.05
	1.5	.337	.94	.453	1.94	.555	2.84	.705	4.17	.818	5.15
	1.0	.450	1.84	.588	3.22	.704	4.35	.862	5.92	.975	7.01
	.5	.695	4.63	.852	6.43	.970	7.79	1.112	9.40	1.208	10.50

HEAD, 1 FOOT

Bottom contraction.	End contractions.	1-foot notch. Velocity of approach.	1-foot notch. Increase of discharge.	1½-foot notch. Velocity of approach.	1½-foot notch. Increase of discharge.	2-foot notch. Velocity of approach.	2-foot notch. Increase of discharge.	3-foot notch. Velocity of approach.	3-foot notch. Increase of discharge.	4-foot notch. Velocity of approach.	4-foot notch. Increase of discharge.
3	2.5	0.132	0.74								
	2.0	.157	.81	0.213	0.82	0.269	0.84	0.342	0.83	0.402	0.87
	1.5	.196	.99	.260	1.08	.317	1.14	.399	1.22	.460	1.29
	1.0	.260	1.40	.337	1.63	.398	1.81	.484	2.06	.543	2.22
	.5	.40	2.94	.477	3.22	.540	3.44	.616	3.72	.661	3.88
2½	2.5	.150	.74								
	2.0	.178	.82	.242	.88	.302	.94	.391	1.04	.460	1.11
	1.5	.224	1.05	.297	1.21	.362	1.34	.461	1.57	.528	1.69
	1.0	.299	1.58	.385	1.89	.457	2.14	.553	2.50	.623	2.76
	.5	.462	3.42	.549	3.73	.625	3.99	.704	4.25	.760	4.48
2	2.5	.175	.73								
	2.0	.209	.84	.284	.97	.352	1.11	.458	1.30	.539	1.42
	1.5	.261	1.13	.348	1.42	.424	1.67	.538	2.01	.620	2.28
	1.0	.353	1.83	.450	2.28	.535	2.63	.648	3.14	.733	3.52
	.5	.538	4.01	.646	4.46	.728	4.80	.829	5.17	.895	5.47
1½	2.5	.208	.74								
	2.0	.252	.94	.341	1.18	.424	1.41	.539	1.71	.638	1.98
	1.5	.314	1.31	.418	1.74	.512	2.12	.648	2.65	.750	3.07
	1.0	.424	2.24	.544	2.87	.646	3.40	.790	4.14	.889	4.68
	.5	.648	4.80	.784	5.53	.885	6.09	1.013	6.77	1.091	7.20
1	2.5	.260	.82								
	2.0	.314	1.12	.427	1.57	.532	2.00	.694	2.69	.810	3.15
	1.5	.385	1.59	.528	[illegible]	.645	2.99	.820	3.91	.952	4.60
	1.0	.525	2.83	.688	3.86	.825	4.73	.999	5.87	1.135	6.77
	.5	.822	6.00	.994	7.29	1.129	8.29	1.298	9.55	1.405	10.37
½	2.5	.350	1.12								
	2.0	.417	1.45	.575	2.40	.720	3.27	.943	4.62	1.120	5.65
	1.5	.530	2.00	.710	3.53	.875	4.73	1.119	6.50	1.308	7.88
	1.0	.716	3.83	.930	5.65	1.118	7.23	1.380	9.49	1.576	11.2
	.5	1.120	8.15	1.37	11.0	1.58	13.3	1.83	16.01	2.01	18.0

TABLE XIV.—*Velocities of approach (in feet per second) and percentages of increase in discharges through Cipolletti notches caused by different bottom and end contractions*

HEAD, 0.6 FOOT

Bottom contraction.	End contractions.	1-foot notch.		1½-foot notch.		2-foot notch.		3-foot notch.		4-foot notch.	
		Velocity of approach.	Increase of discharge.	Velocity of approach.	Increase of discharge.	Velocity of approach.	Increase of discharge.	Velocity of approach.	Increase of discharge.	Velocity of approach.	Increase of discharge.
Feet.	*Feet.*		*Per ct.*		*Per ct.*		*Per ct.*		*Per ct.*		*Per ct.*
1½	2.0	0.158	0.84	0.207	1.02	0.251	1.21	0.321	1.45	0.373	1.61
	1.5	.196	1.11	.255	1.38	.304	1.60	.377	1.95	.429	2.19
	1.0	.260	1.70	.329	2.08	.381	2.36	.454	2.77	.504	3.02
	.5	.400	3.32	.469	3.83	.518	4.20	.580	4.66	.617	4.93
1	2.0	.205	.90	.274	1.25	.331	1.55	.425	2.05	.492	2.39
	1.5	.257	1.20	.335	1.71	.400	2.17	.500	2.82	.569	3.30
	1.0	.344	1.84	.434	2.60	.501	3.17	.607	4.06	.671	4.60
	.5	.529	4.00	.622	4.92	.690	5.61	.770	6.41	.826	6.98
½	2.0	.300	1.11	.399	1.81	.487	2.42	.625	3.40	.725	4.09
	1.5	.377	1.51	.492	2.55	.589	3.44	.737	4.79	.847	5.80
	1.0	.505	2.39	.636	3.93	.750	5.30	.908	7.18	1.013	8.39
	.5	.782	6.03	.932	8.02	1.037	9.43	1.173	11.28	1.263	12.48

HEAD, 1 FOOT

2	2.0	0.250	1.19	0.322	1.22	0.386	1.24	0.488	1.28	0.561	1.30
	1.5	.314	1.52	.397	1.70	.467	1.84	.575	2.08	.648	2.22
	1.0	.422	2.40	.514	2.80	.590	3.15	.698	3.62	.769	3.92
	.5	.655	6.16	.746	6.41	.813	6.61	.896	6.88	.951	7.01
1½	2.0	.300	1.34	.388	1.49	.465	1.61	.590	1.82	.680	1.98
	1.5	.378	1.78	.477	2.10	.562	2.40	.693	2.85	.785	3.17
	1.0	.508	2.89	.622	3.53	.714	4.06	.844	4.79	.937	5.31
	.5	.795	7.29	.906	7.79	.989	8.18	1.094	8.64	1.163	8.95
1	2.0	.374	1.60	.489	2.06	.586	2.44	.758	3.13	.864	3.55
	1.5	.471	2.20	.601	2.92	.710	3.55	.888	4.52	1.003	5.19
	1.0	.643	3.76	.787	4.83	.908	5.73	1.083	7.07	1.200	7.92
	.5	1.010	9.20	1.159	10.28	1.271	11.08	1.410	12.09	1.503	12.72
½	2.5	.64	3.3	.818	4.8	.968	6.07	1.21	8.07	1.391	9.56
	2.0	.508	2.30	.660	3.64	.799	4.87	1.013	6.73	1.202	8.39
	1.5	.640	3.30	.818	4.80	.969	6.09	1.210	8.08	1.391	9.56
	1.0	.864	5.40	1.077	7.58	1.258	9.43	1.505	11.95	1.688	13.81
	.5	1.390	11.89	1.605	14.63	1.782	16.85	2.015	19.80	....	....

Figure 16 shows the variation of the percentages of increase in the discharges through a 1-foot rectangular notch, with heads of 0.6 foot and 1 foot as the ratio of the cross-sectional area of the weir box (A) to the area of the weir notch (a), decreased with the use of different end and bottom contractions. From these curves it will be seen that changing the position of the sides of the weir box and leaving the bottom in a fixed position has a greater effect upon the discharges than leaving the sides fixed and moving the bottom. This indicates that end contractions have more effect upon the discharges than do bottom contractions. With end contractions equal to $2H$ and a bottom contraction equal to $3H$, or end contractions equal to $3H$ and a bottom contraction equal to $2H$, the mean velocities of approach are about one-third foot per second and the discharges with medium to high heads do not agree closer than approximately 1 per cent with the discharges computed by the formula.

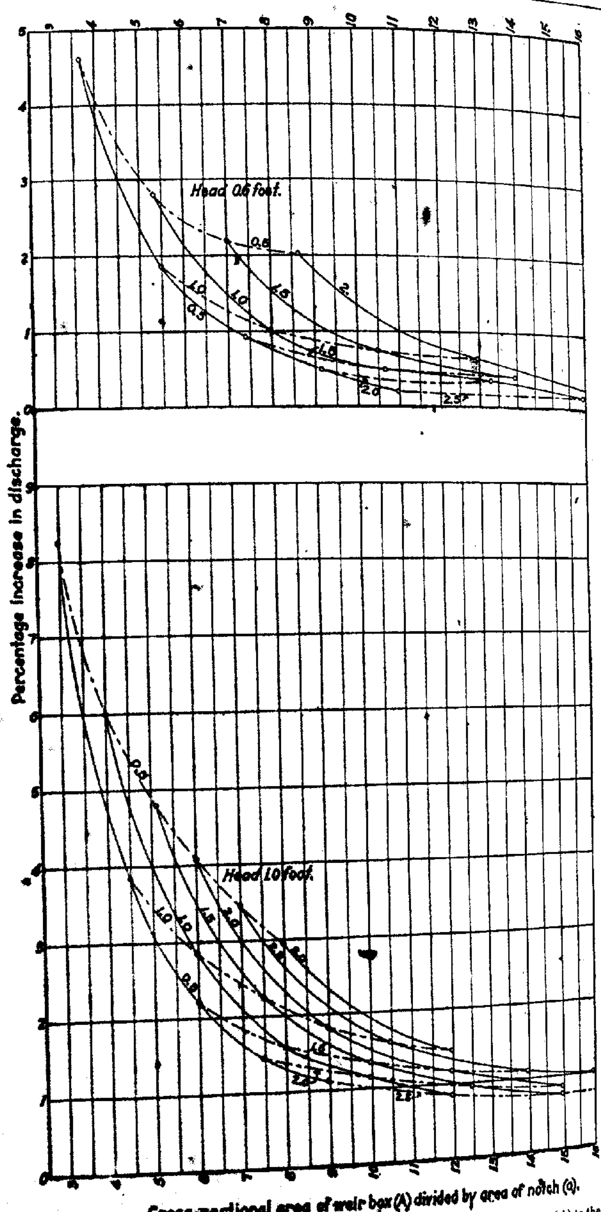

Fig. 16.—Curves showing the effect of different ratios of cross-sectional area of the weir box (A) to the area of the notch (a) upon discharges through a 1-foot rectangular notch with heads of 0.6 foot and 1 foot. [illegible] and contractions in feet.

This indicates that a mean velocity of one-third foot per second is allowable where an error of 1 per cent in discharge is permissible.

By superimposing upon the similar curves for Cipolletti notches the curves showing the effect of different end and bottom contractions upon the discharges through rectangular notches, it was found that the end-contraction distances for Cipolletti notches should be taken from about the middle point of the side of the notch instead of from the end of the crest, in order to make the results of the two types of notches comparable.

Since the minimum bottom and end contractions possible without increasing the discharges beyond an allowable limit increase with the increase of the head run, weir boxes should be designed so as to give discharges within the allowable limit when the highest head intended to be run over the notch is being run. Francis stated (5, p. 72 and 134):

> In order that the end contraction may be complete, the sill and sides of the weir must be so far removed from the bottom and lateral sides of the reservoir (weir box) that they may produce no more effect upon the discharge than if they were removed a distance infinitely great.

He concludes from his experiments than an end contraction of $1H$ and a bottom contraction of $2H$ are the least permissible in order that his formula may apply.

Smith (10, p. 120) gave the necessary end contractions as $3H$. He also suggested (p. 122) that the effect of contraction should not be confused with the effect of velocity of approach, which is so commonly done in taking the term "complete contraction" to include both the effect of contraction and the velocity of approach. Cipolletti (3, p. 23–24) accepted the results of the Francis experiments for end and bottom contractions. He also quotes a rule deduced by Lesbros from results of his (Lesbros's) experiments, that both contractions should be at least 2.7 times the depth of the nappe. Cipolletti (3), from the experiments of Francis (5), deduced the following: (1) When the end contractions equal $2H$ and the bottom contraction $3H$, the bottom and side walls no longer have any appreciable effect upon the discharges through the notch. This condition, he states, may cause an increase of about 0.15 per cent in the discharge. (2) With end contractions of $1.5H$ and a bottom contraction of $2.5H$ the increase in discharge would be about 0.5 per cent. (3) With end contractions of $1H$ and a bottom contraction of $2H$ the discharges will be increased about 1 per cent. He also takes account of the fact that the velocity of approach must not exceed a certain limit.

The ratio of the cross-sectional area of the weir box to the cross-sectional area of the notch necessary for complete contraction has been given by Carpenter (2, p. 29) as 7. The coefficient using this expression of ratio was proposed by J. Weisbach in 1845 and has been elaborated upon by a number of writers and experimenters (6, p. 312). Figure 16 indicates that there is no fixed value of the ratio A to a which will insure

complete contraction in all cases. It also indicates that the value of such ratio should be greater than 7 in all cases, and that 15 probably would come nearer than 7 to meeting average conditions.

EFFECT OF SUPPRESSING BOTTOM CONTRACTIONS WITH A 90° TRIANGULAR NOTCH

In order to throw more light upon the question of the effect of bottom contractions upon discharges through triangular notches (9, p. 114-116) experiments were made with a 90° triangular notch with the floor of the weir box at the same level as the vertex of the notch. The width of the weir box used was 10 feet, being the same as that in the standard test with complete contractions, but in the standard test the floor was about 4½ feet below the vertex of the notch. The discharges through the 90° triangular notch with the bottom contraction entirely suppressed was found to be represented by the formula $Q=2.53H^{2.496}$, which varies but little from Thomson's formula for the flow through a 90° triangular notch having complete bottom contractions. It is probable that some part of the increased discharge obtained when the floor was level with the vertex of the notch was due to the increased velocity of approach. The increase in the discharges amounted to 1.6 per cent with a head of 1 foot, but gradually diminished as the head was decreased. The percentage of increase with heads of 0.3 foot or over is represented by the formula $E=101.6H^{0.016}-100$.

RELATION OF LENGTHS OF NOTCHES TO DISCHARGES

The principal advantage claimed in irrigation practice for Cipolletti notches over other notches has been that the discharges are proportional to the crest lengths. This claim is not in accordance with the limitation put on the notch by Francis and Cipolletti, but has been very generally made in irrigation practice. The failure of this theory is shown in Table XV, in which the discharges through Cipolletti and rectangular notches of different lengths are compared with the discharges through a 1-foot Cipolletti and a 1-foot rectangular notch, multiplied by the number of feet in length of the notches. The percentages in the table represent the failure of the larger notches to give discharges proportional to their lengths. It will be seen from the table that rectangular notches give discharges which are more nearly proportional to their lengths than do Cipolletti notches. The percentages of error increase with the head and length of the crest until the discharge through a 4-foot Cipolletti notch with a 1-foot head is 9.2 per cent less than four times the flow through a 1-foot notch with a 1-foot head, and the discharge through a 4-foot rectangular notch is 4 per cent greater than 4 times the discharge through a 1-foot rectangular notch with a 1-foot head. Side slopes of 1 to 4 are therefore too flat and vertical sides are too steep to give discharges proportional to the length of the crest.

TABLE XV.—*Relation of length to discharge (in cubic feet per second) of weirs*

RECTANGULAR NOTCHES

Head.	1-foot crest.	1.5-foot crest.				2-foot crest.				3-foot crest.				4-foot crest.			
	Discharge.	Discharge.	1.5 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.	Discharge.	2 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.	Discharge.	3 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.	Discharge.	4 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.
Feet.																	
0.20	0.291	0.439	0.437	0.002	0.5	0.588	0.582	0.006	1.0	0.887	0.873	0.014	1.6	1.187	1.164	0.023	2.0
.25	.404	.609	.606	.003	.5	.817	.808	.009	1.1	1.233	1.212	.021	1.7	1.650	1.616	.034	2.1
.30	.527	.796	.790	.006	.8	1.068	1.054	.014	1.3	1.612	1.581	.031	2.0	2.158	2.108	.050	2.4
.40	.804	1.214	1.206	.008	.7	1.630	1.606	.024	1.5	2.464	2.412	.052	2.2	3.299	3.216	.083	2.6
.50	1.113	1.684	1.670	.014	.8	2.262	2.226	.036	1.6	3.421	3.339	.082	2.5	4.583	4.452	.131	2.9
.60	1.453	2.201	2.180	.021	1.0	2.956	2.906	.050	1.7	4.474	4.359	.115	2.6	5.996	5.812	.184	3.2
.70	1.819	2.756	2.729	.027	1.0	3.705	3.638	.067	1.8	5.611	5.457	.154	2.8	7.522	7.276	.246	3.4
.80	2.210	3.351	3.315	.036	1.1	4.506	4.420	.086	1.9	6.828	6.630	.198	3.0	9.158	8.840	.318	3.6
.90	2.624	3.980	3.936	.044	1.1	5.354	5.248	.106	2.0	8.118	7.872	.246	3.1	10.891	10.496	.395	3.8
1.00	3.058	4.642	4.587	.055	1.2	6.247	6.116	.131	2.1	9.476	9.174	.302	3.3	12.716	12.232	.484	4.0

CIPOLLETTI NOTCHES

Head.	1-foot crest. Discharge.	1.5-foot crest. Discharge.	1.5 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.	2-foot crest. Discharge.	2 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.	3-foot crest. Discharge.	3 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.	4-foot crest. Discharge.	4 × discharge through 1-foot notch.	Difference. Amount.	Difference. Per cent.
0.20	0.302	0.450	0.453	—0.003	—0.7	0.599	0.604	—0.005	—0.8	0.898	0.906	—0.008	—0.9	1.198	1.208	—0.010	—0.8
.25	.423	.628	.634	— .006	— .9	.836	.846	— .010	—1.2	1.252	1.269	— .017	—1.3	1.669	1.692	— .023	—1.4
.30	.557	.826	.835	— .009	—1.1	1.098	1.114	— .016	—1.4	1.642	1.671	— .029	—1.7	2.188	2.228	— .040	—1.8
.40	.866	1.277	1.299	— .022	—1.7	1.692	1.732	— .040	—2.3	2.526	2.598	— .072	—2.8	3.361	3.464	— .103	—3.0
.50	1.221	1.793	1.831	— .038	—2.1	2.370	2.442	— .072	—2.9	3.529	3.663	— .134	—3.7	4.691	4.884	— .193	—4.0
.60	1.623	2.371	2.434	— .063	—2.6	3.126	3.246	— .120	—3.7	4.644	4.869	— .225	—4.6	6.166	6.492	— .326	—5.0
.70	2.069	3.006	3.103	— .097	—3.1	3.955	4.138	— .183	—4.4	5.861	6.207	— .346	—5.6	7.772	8.276	— .504	—6.1
.80	2.559	3.700	3.838	— .138	—3.6	4.855	5.118	— .263	—5.1	7.177	7.677	— .500	—6.5	9.507	10.236	— .729	—7.1
.90	3.092	4.448	4.638	— .190	—4.1	5.822	6.184	— .362	—5.9	8.586	9.276	— .690	—7.4	11.359	12.368	—1.009	—8.2
1.00	3.667	5.251	5.500	— .249	—4.5	6.856	7.334	— .478	—6.5	10.085	11.001	— .916	—8.3	13.325	14.668	—1.343	—9.2

For the purpose of throwing light upon what would be the probable shape of a type of notch the discharges through which, with the given head, would be proportional to the crest length, the data obtained for the 2-foot rectangular, the 2-foot Cipolletti, and the 2-foot trapezoidal

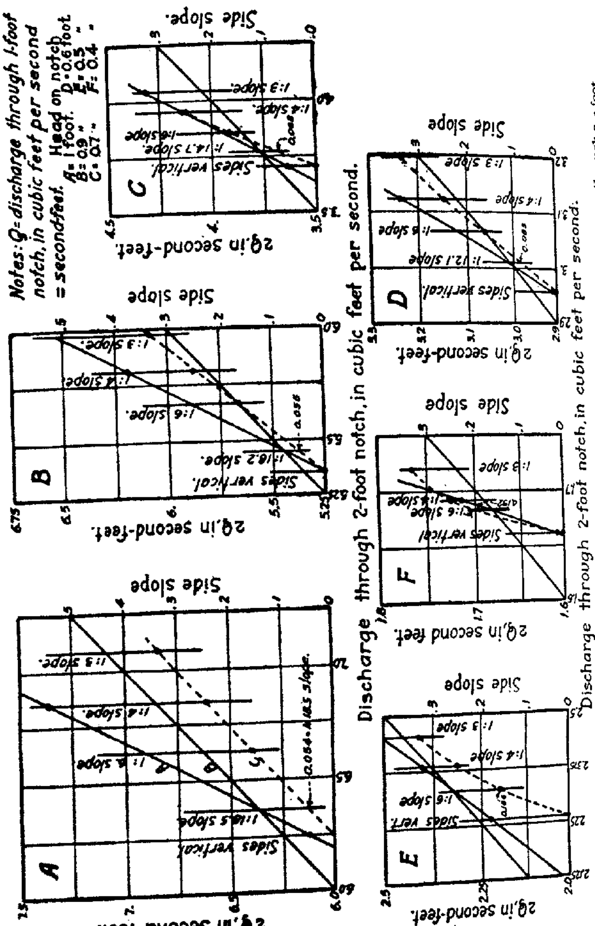

FIG. 17.—Curves showing the side slopes required with different heads in order that the discharge through a 2-foot notch will be twice the discharge through a 1-foot notch.

notches with side slopes 1 to 3 and 1 to 6 were plotted, a set of curves being made for each of the following heads: 0.4, 0.5, 0.6, 0.7, 0.9, and 1 foot (fig. 17). Lines *A* were obtained by plotting the actual discharges through the rectangular and Cipolletti notches with a given head against twice the discharges through 1-foot notches with the same head. Since no experiments were made with 1-foot notches having side slopes

1 to 3 and 1 to 6, it was assumed that similar plottings for such notches would lie on the same straight line as those for the rectangular and Cipolletti notches. Lines *B* pass through the origin and have a slope of 45°. The discharges through a 2-foot notch with the various heads that would fulfill the condition of being twice the discharge through a 1-foot notch with the same head must lie on this 45° line. Curves *C* were obtained by plotting the discharges through the 2-foot notches of different shapes against the decimal expression of the side slope of the notches.

In each set of curves the point of intersection with the *C* curve of a vertical line drawn through the point of intersection of lines *A* and *B* indicates the side slopes which are necessary with a given head in order that the discharge through a 2-foot notch shall be twice that through a 1-foot notch. The slopes found expressed as ratios of the horizontal to the vertical distance are given in Table XVI and indicate that the sides of a 2-foot notch which would give twice the discharges of a similar 1-foot notch with heads up to 1 foot at least must be curves and must approach the vertical as they go up.

TABLE XVI.—*Side slopes necessary in order that a 2-foot notch discharge twice the amount of water from a 1-foot notch*

Head.	Slopes.
Feet.	
1.0	1 to 18.5
.9	1 to 18.2
.7	1 to 14.7
.6	1 to 12.1
.5	1 to 6.5
.4	1 to 5.25
.2	[a] 1 to 4.0

[a] Obtained from data for 0.2 head.

No attempt was made to determine the exact shape of the sides of the notch. They would be so complex, however, that their construction would render impracticable the use of such notches on the farm. Because of the appreciable difference in the effects of contraction with notches of different sizes, a similar comparison of the discharges through larger notches with those through a 1-foot notch would probably give results different from those obtained for the 2-foot notch.

SUBMERGED RECTANGULAR AND CIPOLLETTI NOTCHES

A notch is said to be submerged or "drowned" when the water level on the downstream side is higher than the crest of the notch. To determine the effect of submergence upon the discharges 757 experiments were made with the 1-, 2-, 3-, and 4-foot rectangular and Cipolletti notches used in the free-flow experiments. The conditions on the up-

stream side of the weir were those of the standard weir box—that is, the width of box was 10 feet; the depth of the box 6 feet; and the distance of the floor from the crest of the notches about 4½ feet. A bulkhead was placed across the escape channel of the standard box, parallel to and about 5¾ feet from the plane of the weir, thus making the spill box 10 feet wide, 5¾ feet long, and 4 feet deep, the floor being about 2½ feet below the crest of the notch. The height of the water in the escape channel was controlled by a steel head gate 20 inches square with a vertical slide set in the middle of the bulkhead about 0.5 foot above the floor, and by a 4-inch gate valve set near one end of the bulkhead, the finer regulation being made with this valve. The elevation of the water in the escape channel was determined by a hook gauge set in the concrete gauge box, which was connected with the escape channel by two 1-inch pipes which entered near the floor line 3½ feet from the plane of the weir.

Several minutes were required to adjust the flow of the water before an experiment was started, but when the desired condition of flow had been obtained it was maintained without difficulty throughout the test, except when the head on the upstream side of the weir was high and the head on the downstream side was small. Under this condition the large volume of water flowing through the notch depressed the water surface immediately downstream from the notch. This was followed by a standing wave, and the resulting backlashing and surging in the escape channel caused intermittent pulsations in the hook-gauge still box. The errors, however, were largely compensating, as is indicated by the consistent curves obtained from the experimental data.

The discharges with different heads through the different notches, with free flow and with different depths of submergence, were plotted (figs. 18 to 25) with discharges in cubic feet per second as abscissas and the heads upstream from the weir (H_A) as ordinates. Curves were drawn showing the discharges with different heads upstream from the weir (H_A) with varying differences (H_D) between the head upstream from the weir (H_A) and the head downstream from the weir (H_B). The method of interpolating between the values given on the curves in figures 18 to 25 is indicated by the dotted lines in figure 18 and is based upon the fact that $H_A = H_B + H_D$. The $H_D = 0.15$ line must pass through the points where the various H_B lines intersect the H_A lines and satisfy the equation $H_A - H_B = 0.15$. The $H_B = 0.65$ line would be located similarly upon the points of intersection of the H_A and H_B lines. Interpolations for other depths of submergence can be made in the same manner by drawing H_A lines for other than even 0.05-foot heads. For the purpose of comparison, the free-flow discharge curve is drawn with each set of submergence curves.

A series of experiments was made to determine the effect upon discharges of changing the conditions in the escape channel from free flow

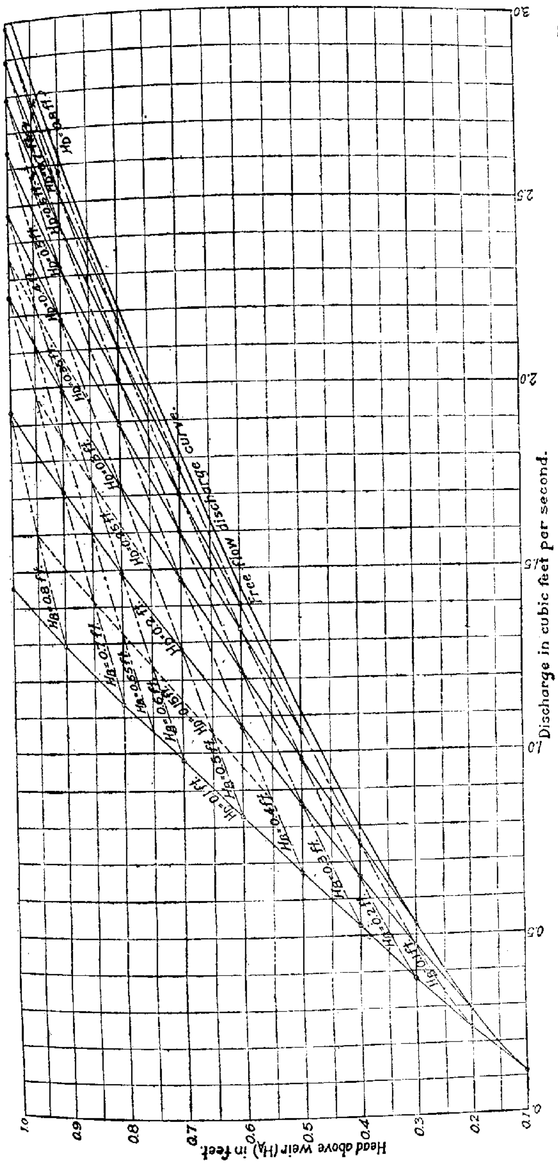

FIG. 18.—Curves showing the discharges through a 1-foot rectangular notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$= effective head.

to submergence. In this set of experiments the head upstream from the weir was made constant, but the conditions downstream were changed by stages in the runs from a free fall of 0.5 foot to a submergence of 0.1 foot. The discharges through this change of conditions remained the same within the limit of the experimental error—0.5 per cent. The

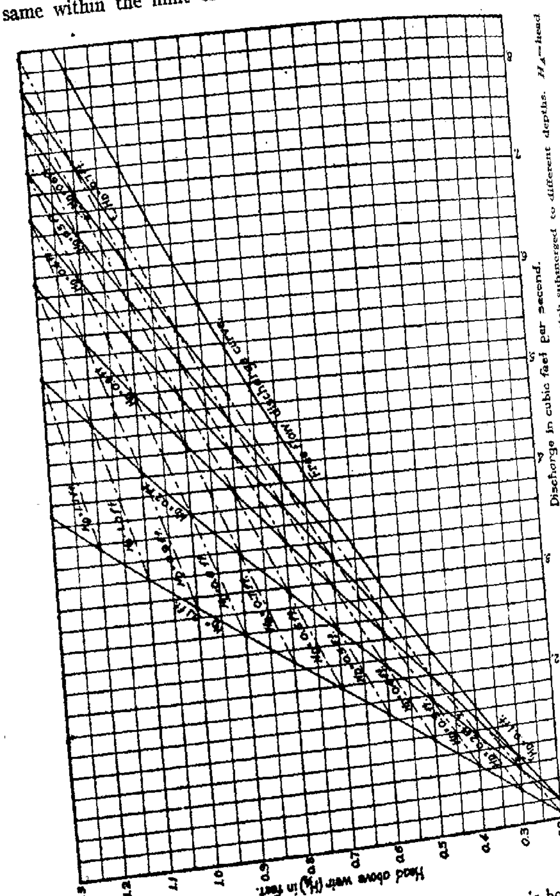

FIG. 19.—Curves showing the discharges through a 2-foot rectangular notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$=effective head.

notches were all thin-edged, the cross section of the weir box in every case was large enough for full-contraction conditions, and the escape channel was wide enough to allow the sheet of water to expand laterally after passing through the notch. In none of the tests was the amount of submergence small enough to make it possible to determine whether the discharge is actually increased with the small amounts of submer-

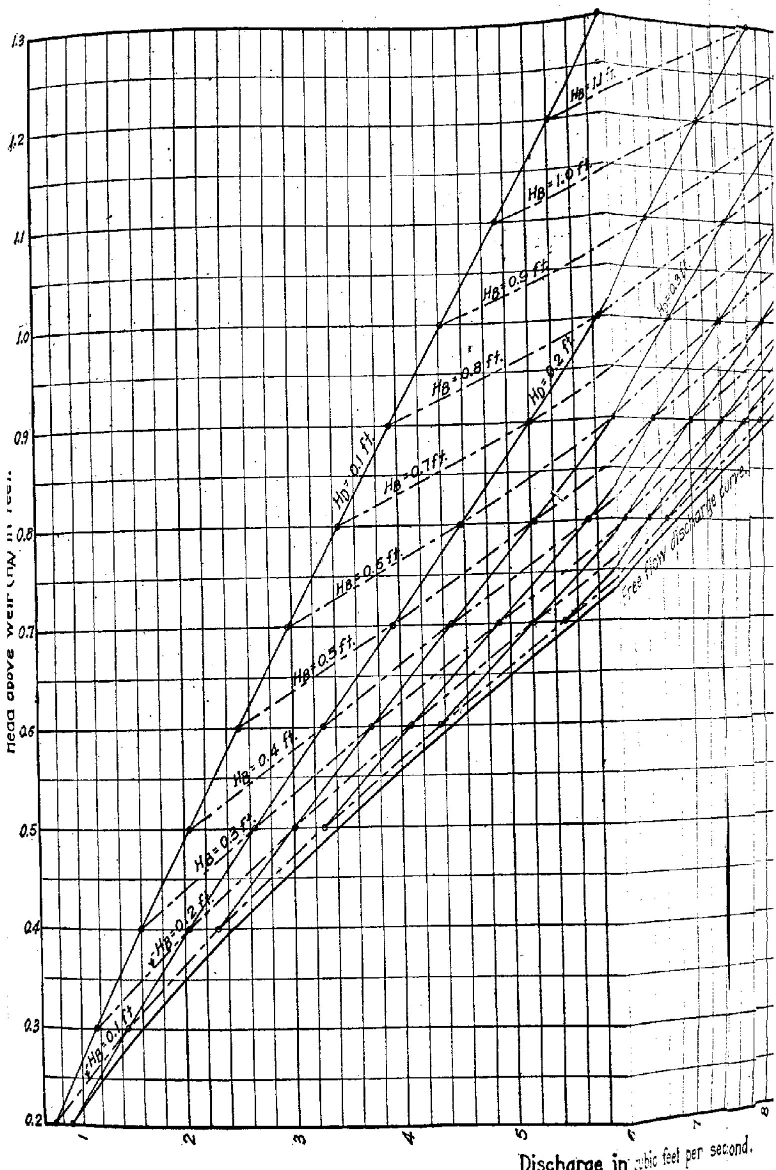

FIG. 20.—Curves showing the discharges through a 3-foot rectangular notch submerged to different depths. H_A= head a

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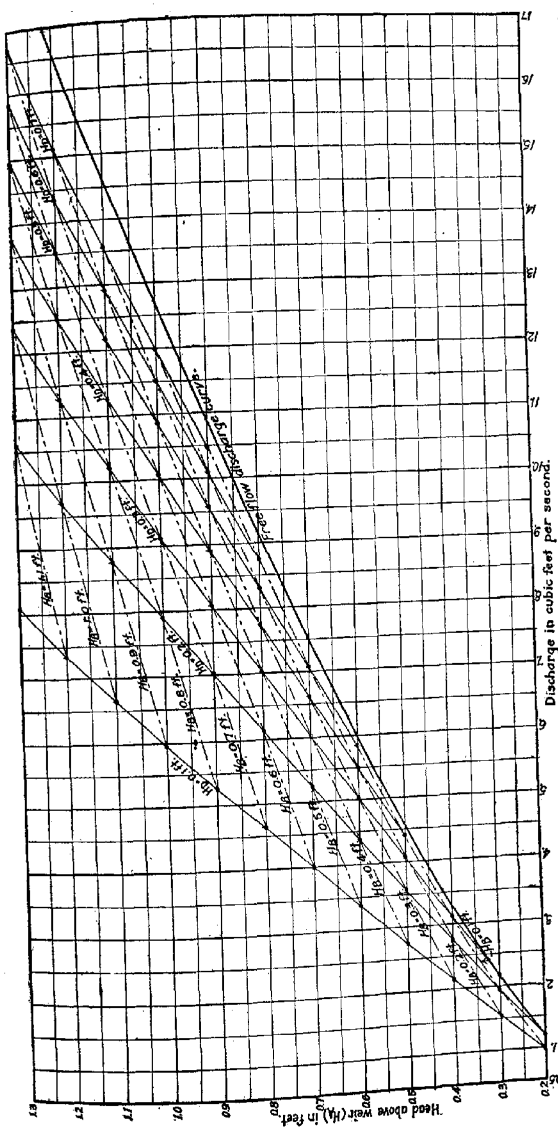

FIG. 21.—Graph showing the discharges through a 4-foot rectangular notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$=effective head.

gence. For all practical purposes, however, it may be stated that the discharge is not materially affected unless the notch is submerged until H_B is at least one-tenth of H_A. When H_B is one-eighth of H_A, the dis-

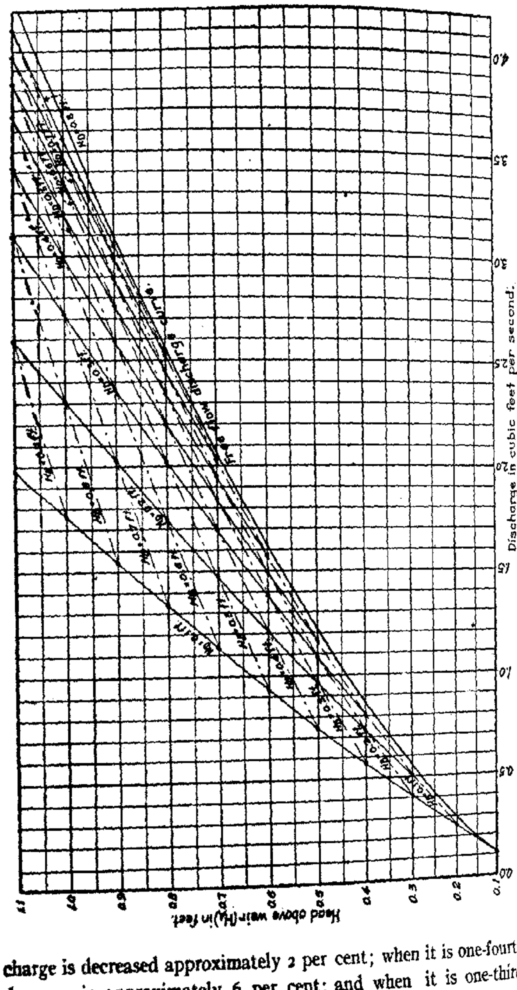

FIG. 22.—Curves showing the discharges through a 1-foot Cipolletti notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$=effective head.

charge is decreased approximately 2 per cent; when it is one-fourth, the decrease is approximately 6 per cent; and when it is one-third, the decrease is approximately 9 per cent. These percentages vary somewhat with the head.

SUMMARY

(1) The discharges through rectangular and Cipolletti notches when plotted logarithmically do not give straight lines and therefore can not be represented correctly by a formula of the type $Q=CLH^n$. It was

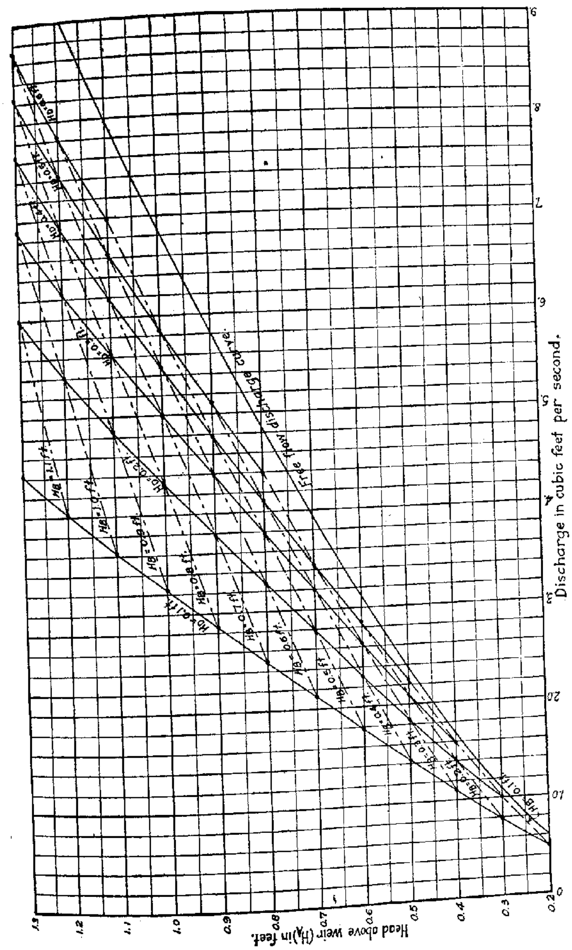

FIG. 23.—Curves showing the discharges through a 2-foot Cipolletti notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$=effective head.

found, however, in the case of the rectangular notches experimented with and the heads of water run, that a straight-line formula could be deduced that within the range of the experiments gave values quite close to the experimental data.

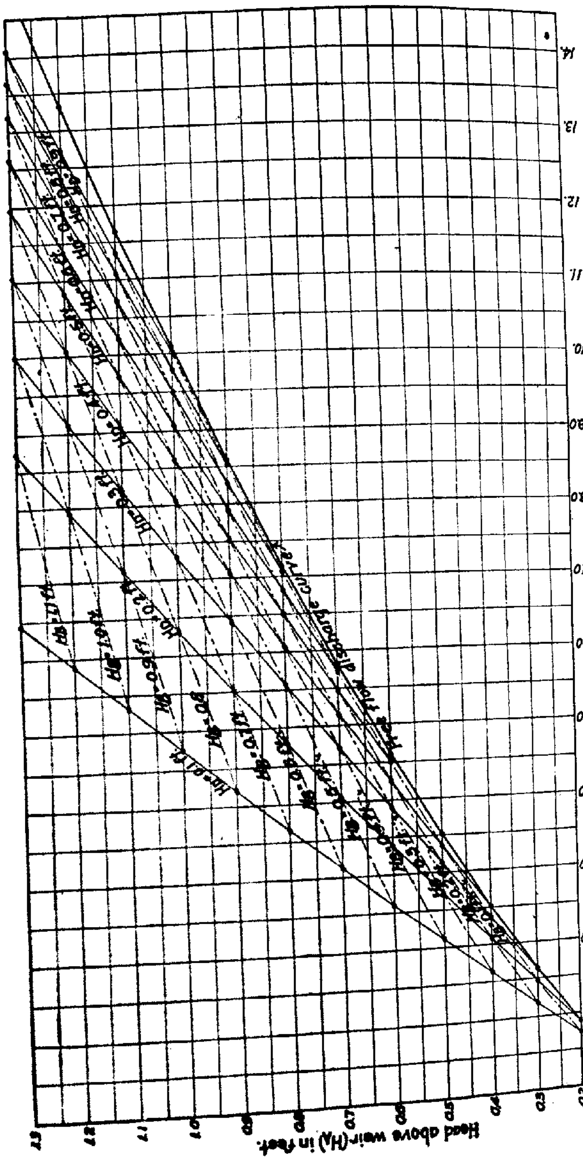

FIG. 24.—Curves showing the discharges through a 3-foot Cipolletti notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$=effective head.

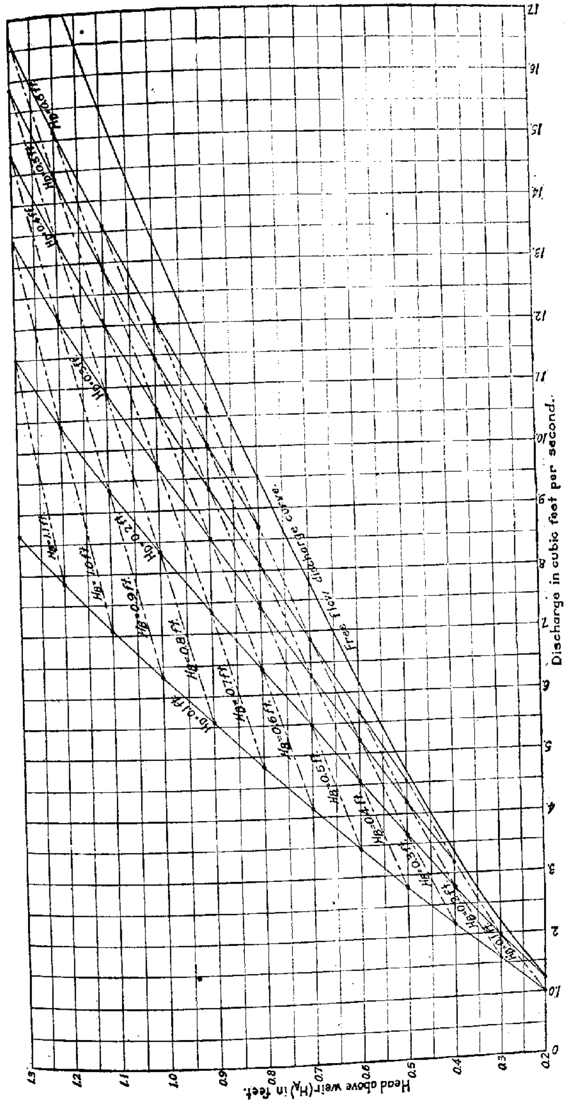

FIG. 25.—Curves showing the discharges through a 4-foot Cipolletti notch submerged to different depths. H_A=head above weir; H_B=head below weir; $H_D=H_A-H_B$=effective head.

(2) The formula

$$Q = 3.247LH^{1.48} - \left(\frac{0.566L^{1.8}}{1+2\ L^{1.8}}\right)H^{1.9}$$

gives discharge values for 1-, 1.5-, 2-, 3-, and 4-foot rectangular notches that agree within a maximum of approximately 1.2 per cent and within an average of 0.28 per cent with the curves plotted from the experimental data.

(3) The discharges through the 0.5-foot rectangular notch do not follow the same law as those for the longer notches. The formula

$$Q = 1.593H^{1.529}\left(1 + \frac{1}{800H^{2.3}}\right)$$

gives values consistent with the curve plotted from the experimental data.

(4) The Francis formula gives values within approximately 2 per cent of the actual discharges, so long as the head does not exceed one-third the length of the notch.

(5) Within the limits of the experiments the formula

$$Q = 3.08L^{1.022}\ H^{(1.46+0.003L)}$$

gives discharge values for the 1-, 1.5-, 2-, 3-, and 4-foot rectangular notches that agree within a maximum of 0.7 per cent, and an average of 0.26 per cent, with the values given in the curves plotted from the experimental data.

(6) The formula $Q = 1.566H^{1.504}$ gives values for the 0.5-foot rectangular notch that agree within 1 per cent with the curves plotted from the experimental data.

(7) The curve-line formula for rectangular notches takes account of the law of variation of the discharge curves better than does the straight-line formula and, consequently, it appears that it will give closer values for higher heads and longer notches than those experimented with.

(8) The formula

$$Q = 3.247LH^{1.48} - \left(\frac{0.566L^{1.8}}{1+2L^{1.8}}\right)H^{1.9} + 0.609H^{2.5}$$

gives discharge values for the 1-, 1.5-, 2-, 3-, and 4-foot Cipolletti notches that agree within 0.5 per cent with the curves plotted from the experimental data, except in the case of the lower heads on the 1-foot notch, where the maximum divergence is approximately 1½ per cent.

(9) The discharges through the 0.5-foot Cipolletti notch do not follow the same law as those for longer notches. The formula

$$Q = 1.593H^{1.529}\left(1 + \frac{1}{800H^{2.3}}\right) + 0.587H^{2.53}$$

represents the discharges through such a notch.

(10) The Cipolletti formula gives discharge values within 1½ per cent of the actual discharges so long as the head does not exceed one-third the length of the crest of the notch.

(11) The formula

$$Q = 3.08L^{1.022}H^{(1.46+0.003L)} + 0.6H^{2.6},$$

which is based on the straight-line formula for rectangular notches, gives discharge values for the 1-, 1.5-, 2-, 3-, and 4-foot Cipolletti notches that agree within a maximum of 1 per cent with the curves plotted from the experimental data, the divergences at all but a few points being 0.5 per cent or less. The formula for the 0.5-foot notch is $Q = 1.566H^{1.504} + 0.56H^{2.55}$.

(12) The Cipolletti type of notch does not give discharges as nearly proportional to the length of crest as does the rectangular type, consequently, since rectangular notches are simpler to construct and the formula for such notch gives as accurate discharge values as does the formula for Cipolletti notches, the rectangular-notch weir is to be preferred.

(13) The general formula for discharges through triangular notches of from 28° 4′ to 90°, and probably up to 109°, is

$$Q = (0.025 + 2.462\ S)H^{\left(2.5 - \frac{0.0195}{S^{0.75}}\right)}$$

where H is the head in feet and S the slope of the sides. Triangular notches having side slopes greater than about 1 to 4 (109°) are impractical, as the nappe adheres.

(14) The 90° triangular notch is the most practical triangular notch and should be used in preference to either rectangular or Cipolletti notches for discharges up to approximately 3 cubic feet per second. The approximate formula $Q = 2.49H^{2.48}$ will give discharge values for 90° notches which agree very closely with the value obtained with the general formula for triangular notches.

(15) The crest and sides of a weir notch need not be knife-edged. They are sufficiently sharp if the upstream corner of the edges is a distinct angle of 90° or less and the thickness of the edges is not so great that the water will adhere to them.

(16) The head should be measured upstream from the weir a distance of at least $4H$, or sidewise from the end of the crest in the plane of weir a distance of at least $2H$.

(17) The distances required for full contractions with rectangular and Cipolletti notches are approximately $2H$, but an additional cross-sectional area of the weir box is required to reduce the velocity of approach.

(18) With end contractions equal to $2H$ and a bottom contraction equal to $3H$, or end contractions equal to $3H$ and a bottom contraction equal to $2H$, the mean velocities of approach are about ⅓ foot

per second, and the discharges with medium to high heads do not agree more closely than approximately 1 per cent with the discharges computed by the formulas.

(19) The average ratio of the cross-sectional area of the weir box (A) to the cross-sectional area of the notch (a) required to give discharges within 1 per cent of the values obtained with the formula is greater than 7 and is probably near 15.

(20) In order to make the results comparable with those for rectangular notches, the end contractions for trapezoidal notches should be measured from about the middle point of the side of the notch, rather than from the end of the crest.

(21) A notch which would give discharges proportional to the lengths of the notches would probably have curved sides, the slope decreasing with the head.

(22) For all practical purposes, discharges through rectangular and Cipolletti notches are not affected until the notch is submerged to a depth equal to one-tenth the head upstream from the weir. Submergence equal to one-eighth the head upstream from the notch decreases the discharge approximately 2 per cent, that equal to one-fourth approximately 6 per cent, and that equal to one-third approximately 9 per cent.

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IDENTITY OF ERIOSOMA PYRI

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This paper has been written in order to reinstate the woolly aphis described by Fitch from apple (*Malus* spp.) roots, to point out its distinctness from the woolly apple aphis (*Eriosoma lanigerum* Hausmann), with which it has been confused, and to place it among the species of the genus to which it properly belongs.

In 1851 Fitch [1] described a woolly aphis under the name "*Eriosoma pyri*." At the same time he described the work of what seems to be *E. lanigerum* Hausmann on apple. At the time of his original description Fitch evidently did not know of the genus Pemphigus. This is indicated from his remarks in his first report,[2] for in the description in this publication he is quite positive in placing his species in that genus. The description of the wingless forms agrees well, however, with *lanigerum*.

The identity of *pyri* has for many years been in doubt, and the name has been referred to different species as a synonym. The writer,[3] in his recent work on the woolly aphis, considered it to be *lanigerum*. This was based on two things: The description of the wingless forms, with the possibility of abnormality in the winged form, and Gillette's [4] statement in regard to the type. One fact, however, seems evident. The descriptions given by Fitch for his winged forms could not have been made from normal migrants of *lanigerum*. In fact, they could not have been made from winged forms of *lanigerum* at all. This is particularly true of the description in the first report.

Fitch's original notes on the species are now in the writer's hands, and they throw some interesting light on the question. After describing the wings minutely, Fitch says: "The wings serve best to distinguish this species, and an exact figure of one or both of them will be the best illustration of it that can be given," and again, "Neuration of the wings identical with that of *Myzoxylus imbricator*." By 1871 Fitch had some feeling that his *pyri* might be a synonym of *lanigerum*, for in his notebook, under October 11 of that year, he suggests such a possibility. He adds, "My winged *lanigera* from Dr. Signoret is a Pemphigus, the 3rd vein being simple, but not so abortive at its base, and has all the veins slender."

[1] Fitch, Asa. Catalogue with references and descriptions of the insects collected and arranged for the State Cabinet of Natural History. *In* 4th Ann. Rpt. [N. Y.] State Cab. Nat. Hist., p. 68. 1851.

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This much remains: Fitch was not sure that he was not dealing with a compound species in his apple-root form and his winged forms. This is shown by the following note: "Amyot describes *Eriosoma lanigerum* as producing excrescences. Can these small lice be that species, and the winged ones another species accidentally present with them?"

What Fitch suspected is, the writer believes, true, and Fitch described the winged form of one species and the work of wingless *lanigerum*.

In the United States National Museum collection there is some material labeled "*P. pyri* Fitch, Type," and mounted by Pergande from the Fitch collection. This proves to agree in every detail with the different descriptions of the winged forms given by Fitch. There seems good reason to believe that the material represents the specimens from which Fitch drew up his diagnosis. This is strengthened by the fact that the species occurs in the vicinity of Washington, D. C., and Vienna, Va., upon apple and upon pear (*Pyrus* spp.) roots. It is particularly common upon pear roots, and it occurs also upon *Crataegus* spp. and ash (*Fraxinus* spp.).

Since this material seems to settle finally the standing of *pyri*, a description is here given of the form based upon this material and upon other specimens collected mostly from pear roots. The form proves to belong to the genus Prociphilus, and in order to separate it from other species of the genus, descriptive notes and figures are given of the other species known to the writer. Particular stress is laid in these notes on the dorsal wax plates of the thorax, since these seem to prove good diagnostic characters.

The writer has never seen specimens of *Prociphilus crataegi* Tullgren, and it may be possible that *pyri* and *crataegi* are the same, since the sensory characters are similar. There seems, however, to be considerable difference in measurements. The question as to their distinctness or identity can only be determined by a careful comparison of the two.

It is possible, also, that *venafuscus* Patch may prove to be *pyri*. But in the specimens studied by the writer the sensoria are much more even, and *pyri* seems to lack the small, pointed projection near the base of the third segment of the antennæ.

The following description will, however, serve to place *pyri*:

Prociphilus pyri (Fitch)

Fall migrant (fig. 1, *E*, *Q*).—**Morphological characters**: Antennal segments as follows: I, 0.064 mm.; II, 0.096 mm.; III, 0.544 mm.; IV, 0.224 mm.; V, 0.24 mm.; VI, base 0.192 mm., unguis 0.064 mm.; segments III to VI with transverse sensoria, usually very irregular in disposition and giving the segments, particularly segment III, a gnarled appearance; segment III with 28 to 35 sensoria, segment IV with 8 or 9, segment V with about the same number, and segment VI with 3 to 6. These sensoria are on the underside of the antennæ, the upper surface being armed with a few hairs situated on tubercles. Head above with two oval or almost circular transparent wax plates. Dorsum of thorax with a pair of rather small, somewhat triangular wax plates. Forewings 4.38 mm. long and 1.43 mm. wide at their greatest width. Hind tibiæ 1.2 mm. long. Length from vertex to tip of cauda, 2.48 mm.

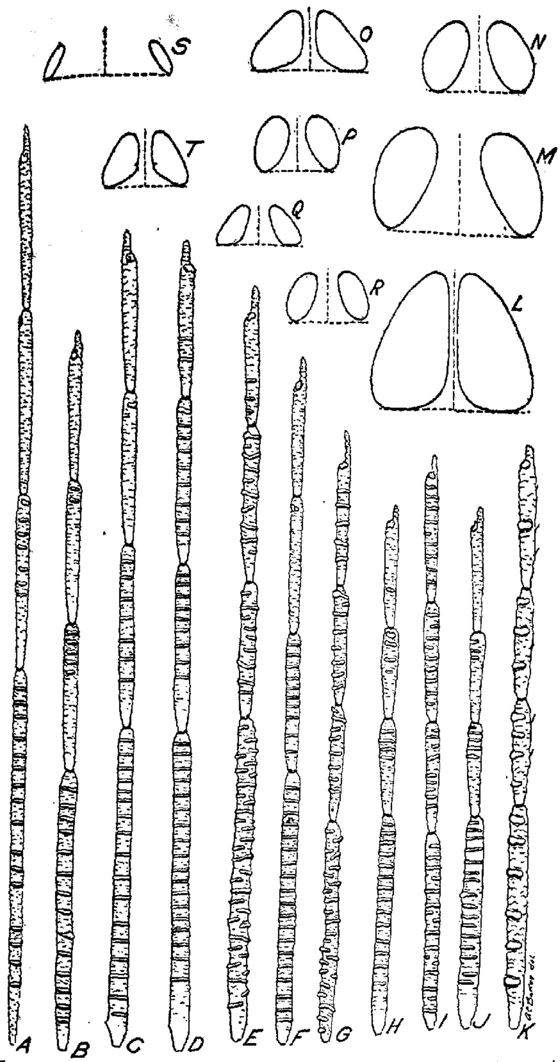

FIG. 1.—Structural characters of the species of Prociphilus. *A*, *P. bumulae:* Distal segments of antenna of spring migrant. *B*, *P. poschingeri:* Distal segments of antenna of spring migrant. *C*, *P. venafuscus:* Distal segments of antenna of spring migrant. *D*, *P. venafuscus:* Distal segments of antenna of fall migrant. *E*, *P. pyri:* Distal segments of antenna of fall migrant. *F*, *P. xylostei:* Distal segments of antenna of spring migrant. *G*, *P. populiconduplifolius:* Distal segments of antenna. *H*, *P. corrugatans:* Distal segments of antenna of spring migrant. *I*, *P. corrugatans:* Distal segments of antenna of spring migrant. *J*, *P. alnifoliae:* Distal segments of antenna. *K*, *P. tessellatus:* Distal segments of antenna. *L*, *P. bumulae:* Thoracic wax plates. *M*, *P. poschingeri:* Thoracic wax plates. *N*, *P. xylostei:* Thoracic wax plates. *O*, *P. venafuscus:* Thoracic wax plates. *P*, *P. corrugatans:* Thoracic wax plates. *Q*, *P. pyri:* Thoracic wax plates. *R*, *P. alnifoliae:* Thoracic wax plates. *S*, *P. populiconduplifolius:* Thoracic wax plates. *T*, *P. tessellatus:* Thoracic wax plates.

Color characters: Eyes, antennæ, and legs black; head black; prothorax and abdomen dull olive green with darker green marginal patches on the abdomen. Thoracic lobes and sternal plate black. Wing veins dark, with dusky bordering; the entire wing often more or less smoky. Head and thorax with a bluish white bloom; abdomen with a long cottony secretion, most pronounced caudad.

Prociphilus aceris (Monell).

Specimens of this species have a pair of large circular wax plates upon the head, and the dorsal wax plates of the thorax are of the same size and shape as those of *venafuscus* Patch. The sensoria on the third segment of the antennæ are oval in shape, some almost circular. They are thus not typical for the genus, but approach those of *attenuatus* Osborn and Sirrine for which Dr. E. M. Patch, of the Maine Experiment Station, has erected the genus Neoprociphilus. There seems to be, however, a gradual gradation from the type to this species. The wing also suggests that of *attenuatus*, and there is some doubt in the writer's mind in regard to the distinctness of Neoprociphilus. The measurements of antennal segments are as follows: III, 0.416 mm.; IV, 0.256 mm.; V, 0.24 mm.; VI, base 0.272 mm., unguis 0.048 mm.

Prociphilus alnifoliae (Williams) (fig. 1, *J*, *R*).

Alnifoliae is a species of medium size with rather short antennæ. The sensoria do not, as a rule, extend entirely across the segments, and they are often acute at each end, thus touching the margins of the segments as a point. The dorsal wax plates of the thorax are quite similar to those of *corrugatans*, being small and oval.

Prociphilus bumulae (Schrank) (fig. 1, *A*, *L*).

This species is very large and the sensoria of the antennæ are even and do not usually extend beyond the margins of the segment. The dorsal wax plates of the thorax are large and triangular and situated close together. In some specimens they almost touch along the median line. The measurements of antennal segments are as follows: III, 0.704 mm.; IV, 0.32 mm.; V, 0.32 mm.; VI, base 0.288 mm., unguis 0.064 mm.

Prociphilus corrugatans (Sirrine) (fig. 1, *H*, *I*, *P*).

This insect is a rather small species with regular sensoria present on the antennæ of the spring migrant, but with them irregularly arranged on the antennæ of the fall migrant. The dorsal wax plates of the thorax are small and oval in outline. The measurements of the antennal segments are: III, 0.32 mm.; IV, 0.144 mm.; V, 0.16 mm.; VI, base 0.128 mm., unguis 0.032 mm.

Prociphilus fraxini-depetalae (Essig).

This species appears to be a synonym of *venafuscus* Patch.

Prociphilus imbricator (Fitch).

This well-known species has not been figured. The sensoria of the antennæ are rather large, approaching those of *tessellatus* (Fitch). The dorsal wax plates of the thorax are small and well separated. The measurements of antennal segments are as follows: III, 0.368 mm.; IV, 0.176 mm.; V, 0.176 mm.; VI, base 0.192 mm., unguis 0.048 mm.

Prociphilus populiconduplifolius (Cowen) (fig. 1, *G*, *S*).

The antennæ of this species are characteristic in that the sensoria extend past the edges of the segments and give them an irregular or beaded effect on the margins. The wax plates on the thorax are also very characteristic, being minute and very widely separated. The antennal measurements are as follows: III, 0.4 mm.; IV, 0.288 mm.; V, 0.208 mm.; VI, base 0.208 mm., unguis 0.064 mm.

In the writer's opinion there is not sufficient difference for the retention of the genus Thecabius. The habits of the stem mothers may be different, as indicated by *patchii* Gillette, and yet the insects are very close in structure. The wax plates and sensoria vary greatly within the genus.

Prociphilus poschingeri (Holzner) (fig. 1, *B*, *M*).

Placed usually as a synonym of *bumulae* Schrank, this form as represented by our specimens shows some differences. The insects are considerably smaller and the dorsal wax plates of the thorax are not triangular and close together as are those of *bumulae*, but are considerably separated and oval in outline. Measurements of antennal segments: III, 0.496 mm.; IV, 0.246 mm.; V, 0.246 mm.; VI, base 0.224 mm., unguis 0.048 mm.

Prociphilus tessellatus (Fitch) (fig. 1, *K*, *T*).

The antennæ of *tessellatus* are hardly typical for this genus. The species seems, however, to fit here as well as anywhere. The sensoria on the antennæ are very broad for the genus and the shape of the segments is not typical. The dorsal wax pores are, however, quite normal. They are somewhat triangular in shape and are somewhat smaller than those of *venafuscus*. In many specimens each is armed with a small hair. Measurements of antennal segments: III, 0.4 mm.; IV, 0.171 mm.; V, 0.171 mm.; VI, base 0.197 mm., unguis 0.032 mm.

Prociphilus venafuscus (Patch) (fig. 1, *C*, *D*, *O*).

The form described by Dr. Patch[1] is the most typical American species and the antennal characters are very similar to those of *bumulae* Schrank. The clouding of the wings met with in *venafuscus* is present also in our specimens of *poschingeri* though it is not noted in those of *bumulae*. The dorsal wax plates of the thorax are, in *venafuscus*, triangular like those of *bumulae*. They are, however, very much smaller. Measurements of antennal segments: III, 0.56 mm.; IV, 0.288 mm.; V, 0.288 mm.; VI, base 0.224 mm., unguis 0.049 mm.

Prociphilus xylostei (De Geer) (fig. 1, *F*, *N*).

Specimens of this species are much smaller than those of *bumulae* or even those of *venafuscus*. The antennal characters are very similar to those of *venafuscus*. The dorsal wax plates of the thorax are, however, of quite different shape in the two species, although they are almost equal in size. Measurements of antennal segments: III, 0.48 mm.; IV, 0.24 mm.; V, 0.24 mm.; VI, base 0.197 mm., unguis 0.048 mm.

The average number of sensoria on the antennæ of the species figured is shown in the illustration. The number varies somewhat in different individuals.

[1] Patch, Edith M. Aphid pests of Maine. *In* Maine Agr. Exp. Sta. Bul. 202, p. 174. 1912.

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